

## **Ph.D. Thesis Defence**

*Julian A. de Marchi, N.S.F. Graduate Fellow*  
Dept. of Mech. Eng., Aero. Eng. & Mechanics  
30 October 1998

*Prof. Dr. Kevin C. Craig, Committee Chair*  
*Prof. Dr. C. James Li, Committee Member*  
*Prof. Dr. Daniel F. Walczyk, Committee Member*  
*Prof. Dr. James Napolitano, Committee Member*

A Summary of Doctoral Research Presented to the Graduate Faculty  
of Rensselaer Polytechnic Institute  
in Partial Fulfillment of the Requirements for the Degree of  
DOCTOR OF PHILOSOPHY  
in Mechanical Engineering

The original of the complete thesis is on file  
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# Ph.D. Thesis Defence

by Julian A. de Marchi, N.S.F. Graduate Fellow

Prof. Dr. Kevin C. Craig, Committee Chair  
Prof. Dr. C. James Li, Committee Member  
Prof. Dr. Daniel F. Walczyk, Committee Member  
Prof. Dr. James Napolitano, Committee Member

Department of Mechanical Engineering, Aeronautical Engineering & Mechanics



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# Personal and Professional Experience

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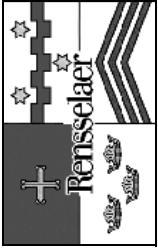
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Julian A. de Marchi

# Academic Profile

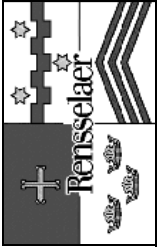
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## • 3-2 Program

- BA: Bard College
- BS: Columbia University SEAS

## • Rensselaer Polytechnic Institute

- Master's Degree, Doctoral Research
- Teaching Assistant for Mechatronics, Mechatronic System Design, Mechatronic Project Management, and Machine Dynamics
- Research Assistant for U.S. Army Research Office and the National Science Foundation
- Lab demonstrations for numerous professional site visits
- Assistance with numerous additional mechatronics projects



# Research Expertise

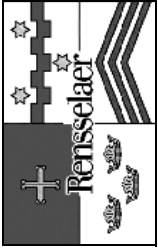
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## • Academic Experience

- mechanics/dynamics modeling & simulation
- control theory
- microprocessors & interface electronics
- programming
- computer & microcontroller interfacing
- real-time system implementation

## • Professional Consultations

- Rensselaer Mechatronics Research Laboratory, in collaboration with Xerox Mechanical Engineering Sciences Laboratory (1998)
- American Guidance Navigation and Control Corporation (1997)
- G.E. Research & Development Center (1995)



# Invited Presentations

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- inverted pendulum *IFAC 1997 - Mechatronics 1998*

DMRAC, mechatronic system design for education

- machine tools *CCAM 1997*

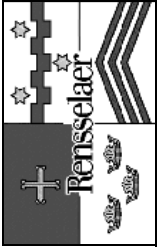
overview of mechatronic issues in machine tool (re)design

- education *ICRAM 1995 - ViCAM 1996*

mechatronic system design approach & curriculum at Rensselaer

- vibration control *WAM 1995*

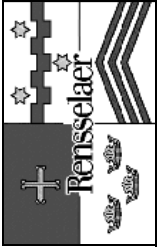
mechanical positioning test bed with flexible "piezoelectric" beam



# Professional Publications

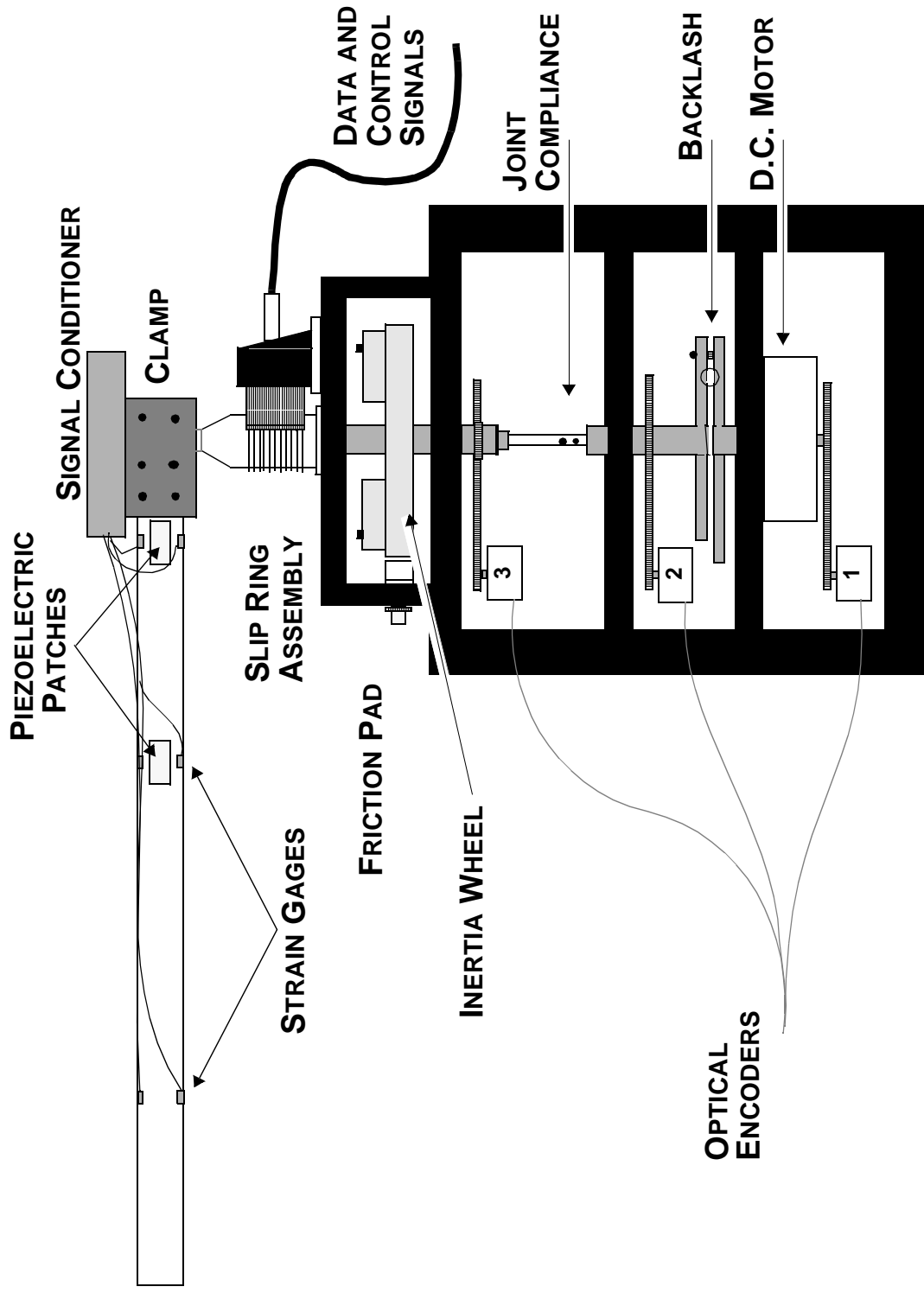
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- published J. of Comp. App. in Eng. Education (Jan. 1996)  
"Mechatronic System Design at Rensselaer."
- for submission — November / December 1998
  - "A symmetric Kinetic + Viscous Friction Identification using the Extended Logarithmic Decrement Method."
  - "The Parametric Harmonic Oscillation Method for Overdamped System Identification."
  - "A Dynamic Backlash Model including Elastic Impact Effects."
  - "Comments on 'Natural Frequencies and Dampings Identification using Wavelet Transform: A pplication to Real Data'."
  - "Swing-Up Control of an Inverted Pendulum."

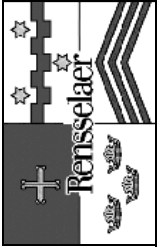


# AGNC Corp: Mech. Pos. System

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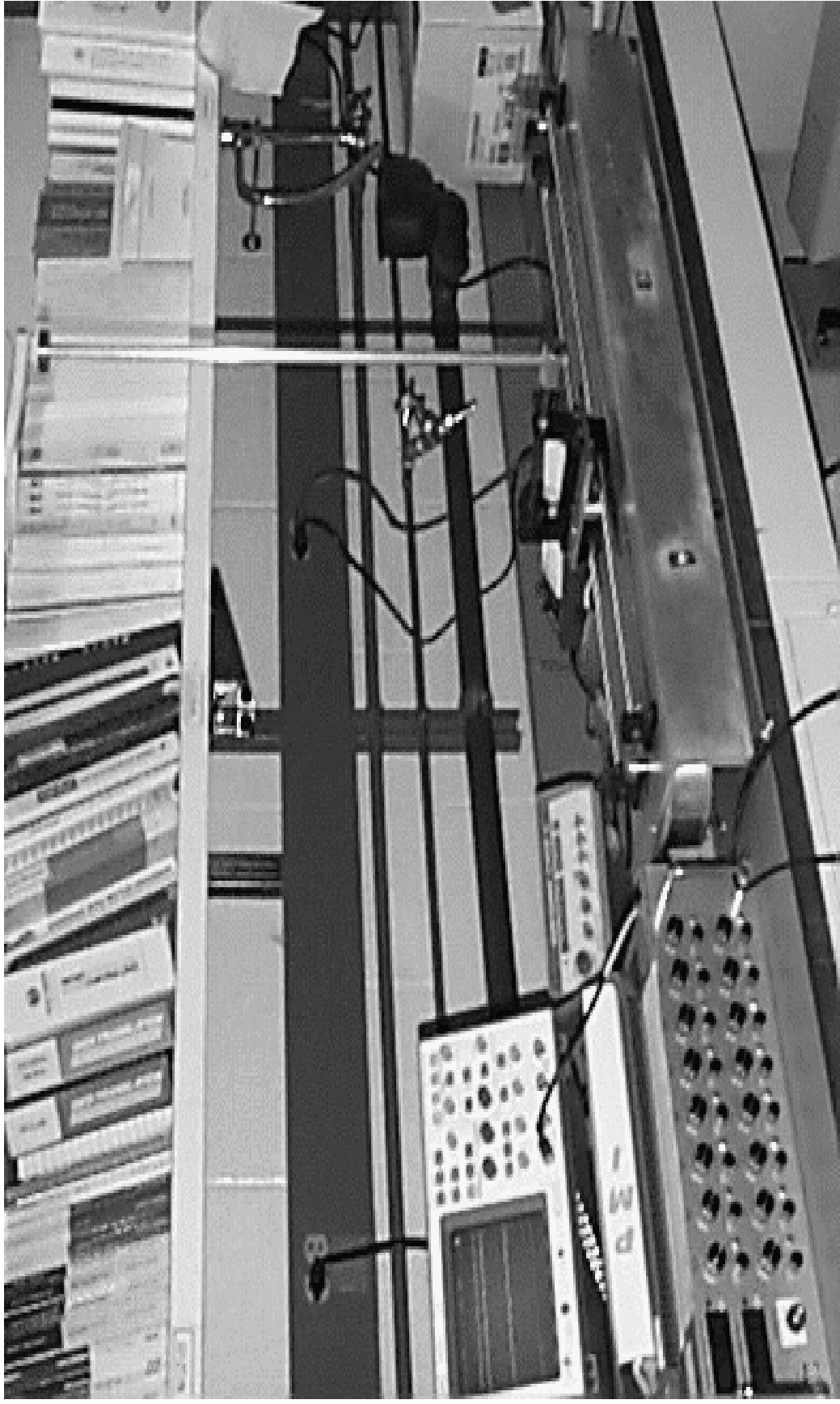




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# RPI: Inverted Pendulum System

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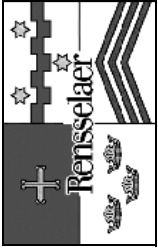
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# Research Overview

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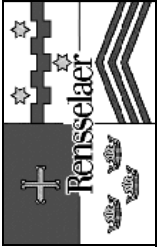
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# Unique Contributions

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- **New** Dynamic Backlash Model
  - state-of-the-art backlash model augmented to include impact dynamics
- **New** Asymmetric Kinetic + Viscous Friction ID
  - asymmetric friction estimation using only the free oscillation response
  - extension of traditional and easy-to-use logarithmic decrement method
- **New** Parametric Harmonic Oscillation ID
  - experimental method using P+D feedback to achieve parametric free vibration response in overdamped systems
  - method allows use of log decrement, Hilbert and wavelet analysis without requiring a sine-swept forced harmonic oscillation
  - produces estimates of inertia and mass based system parameters



# Objectives Achieved

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- Modeling

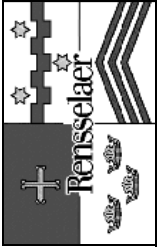
- analytical models of nonlinear subsystems developed
- analytical model of fully-coupled subsystems in simulation

- System ID

- traditional methods extended to asymmetric frictional system
- nonlinear Hilbert and Wavelet methods verified on friction

- Actual machine dynamics

- system ID techniques corroborate published parameter values



# Contribution Highlights

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## • Background work

- thoroughstate-of-the-art review and guidance for future work
- detailed comparative analysis and modeling of nonlinearities
- test bed nonlinear dynamics simulation programme (c code)
- pendulum system improvement & integration
- verification of results by Feeny & Liang (+ Matlab code)
- verification of results by Braun & Feldman (+ Matlab code)
- real-time data-acquisition kernel development

## • Important Contributions

- verification of results by Prof. Dan Walzcyk
- corrections to work by Ruzzene et alii
- extension of work by Ruzzene et alii



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# Doctoral Research: Friction, Backlash and Compliance Modeling and ID

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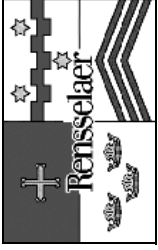
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# Research Motivation

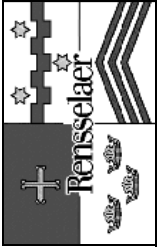
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- Drive train nonlinearities cause machining errors
  - cause of positioning error in workpiece cutting and finishing
  - exacerbated when machining at high speeds and cutting forces
  - cause of damage to workpiece and cutting tools

- Frictional errors
  - thermal expansion
  - stick-slip friction (stiction)

- Backlash errors
  - tolerance reduction
  - impact vibrations
  - pitting damage

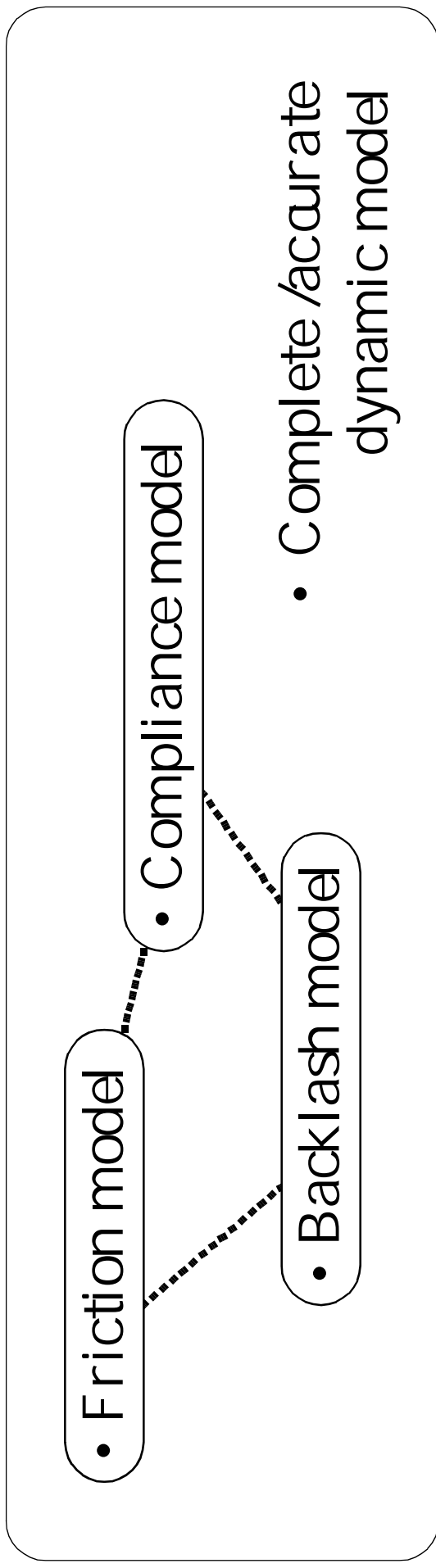
- Compliance error
  - vibration
  - energy storage and release



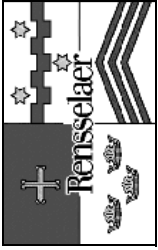
# Research Objectives

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- Using a generalized model of drive nonlinearity, machine tools may be controlled more precisely and accurately
  - model nonlinear friction, backlash and compliance
  - model dynamic interaction between the component nonlinearities
  - apply model to, and verify using data from actual machine tools







# The Overall Approach

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- Develop analytical model
  - model nonlinear friction, backlash & compliance dynamics

- Develop system identification
  - verify system ID procedure on simulated dynamics

- Identify machine tool dynamics
  - test system ID on test bed and actual machine tool
  - corroborate known parameter values and published results



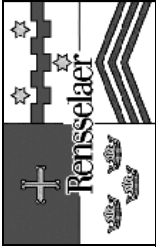
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# Friction Model

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# Friction Details

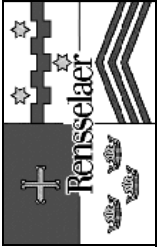
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- Static phenomena
  - normal friction (increases with normal load)
  - rising friction (increases with static contact time)

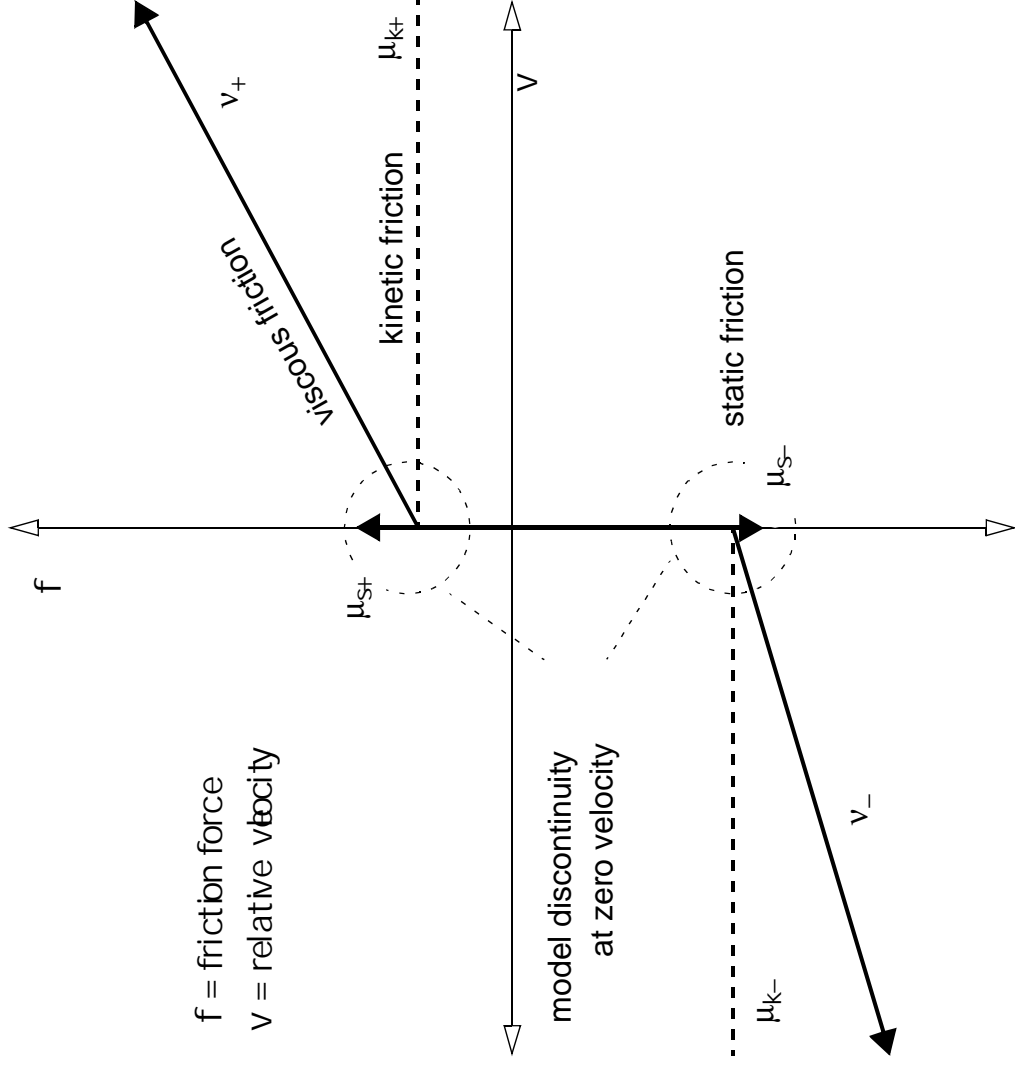
- Hysteretic phenomena
  - presliding friction (hysteretic, static spring force)
  - frictional memory (hysteretic, kinetic spring force)

## • Nonlinear friction

- Dynamic phenomena
  - Stribeck friction (describes transition from static to kinetic)

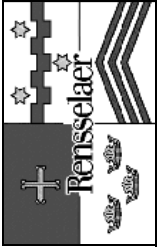


# The Classical Friction Model



- da Vinci (c. 1500) Amontons (1699), Parent (1704, Morin (1834), Reynolds (1884)

- friction depends on the load and the rubbing materials
- friction independent of apparent area of contact and surface finish
- usually static > kinetic friction
- static friction increases with dwell time

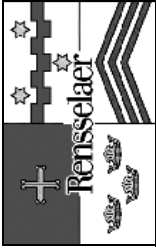


# Dry (Coulomb) Friction

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- "Dry" Static + Kinetic Friction

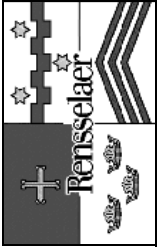
- friction thought to be caused by microscopic interfacial asperities
- static friction acts as a constraint force at zero velocity
- kinetic friction acts against direction of motion at non-zero velocity
- the static friction is almost always greater than the kinetic friction
- therefore, low velocities can result in stick-slip friction (stiction)
- stiction is a cyclical phenomenon when the applied force is constant
- when the force is dynamic, stiction can exhibit chaotic behaviour
- static friction depends on the time at rest (the "dwell" time)
- after an object comes to rest, the friction rises sigmoidally with time from the baseline kinetic to the final static value (rising friction)
- kinetic friction may lessen with higher velocities and steady or extended operation times (temporal Stribeck effect)



# Wet (Viscous) Friction

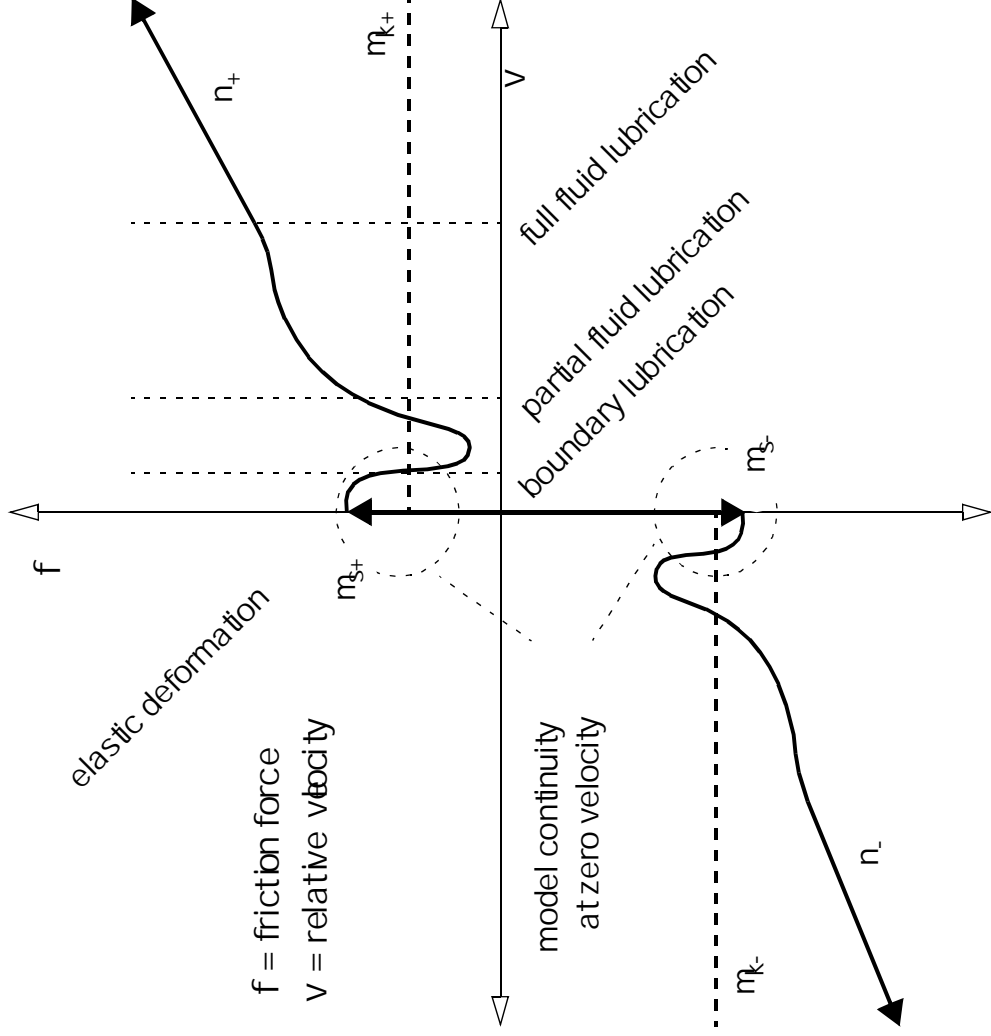
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- “Wet” Kinetic Friction
  - discovered by Reynolds c.1886
  - friction evident in lubricated interfaces like bearings
  - friction is a result of shearing forces in the lubricant
  - friction increases with speed (rate of lubricant shear)
  - under continuous operation, increasing temperature from friction lowers the lubricant viscosity, and thus the friction (Stribeck effect)
  - higher bearing loads also increase the viscous friction, because the lubricant at the interface is squeezed out (“normal” friction)
  - bearing friction under Hertzian contacts (an area rather than a line contact) is further a function of the dynamic interface geometry
  - viscous friction depends on fluid viscosity, which varies with bearing pressure and lubricant temperature

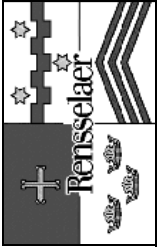


# The Modern Friction Model

• 20<sup>th</sup>-Century Tribologists



- describes transition from static to kinetic regimes, and static phenomena
- friction increases with contact pressure and speed
- friction may differ with the direction of relative motion (asymmetry)
- provides functional continuity across the zero-velocity boundary



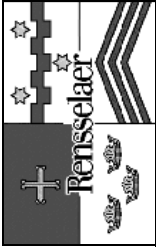
# Phenomenological Approach

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## • The Asperity (Adhesion) Theory

- protruding microscopic surface asperities (“junctions”) interlock rubbing surfaces
- friction increases with the load distributed over the real area of contact (surface finish is irrelevant)
- the real area of contact is related to the contact pressure, not just the load or apparent area of contact
- the pressure over this real area is borne by the asperities themselves, hence localised plastic deformations cause microscopic welding (adhesion) and also trap small pockets of lubricant under high pressures (causing rising friction as the lubricant is squeezed out)
- the asperities cyclically break off and re-adhere (so-called “third-body particles”)
- this theory motivates the modern “bristle” model of dynamic friction





# Rising Friction

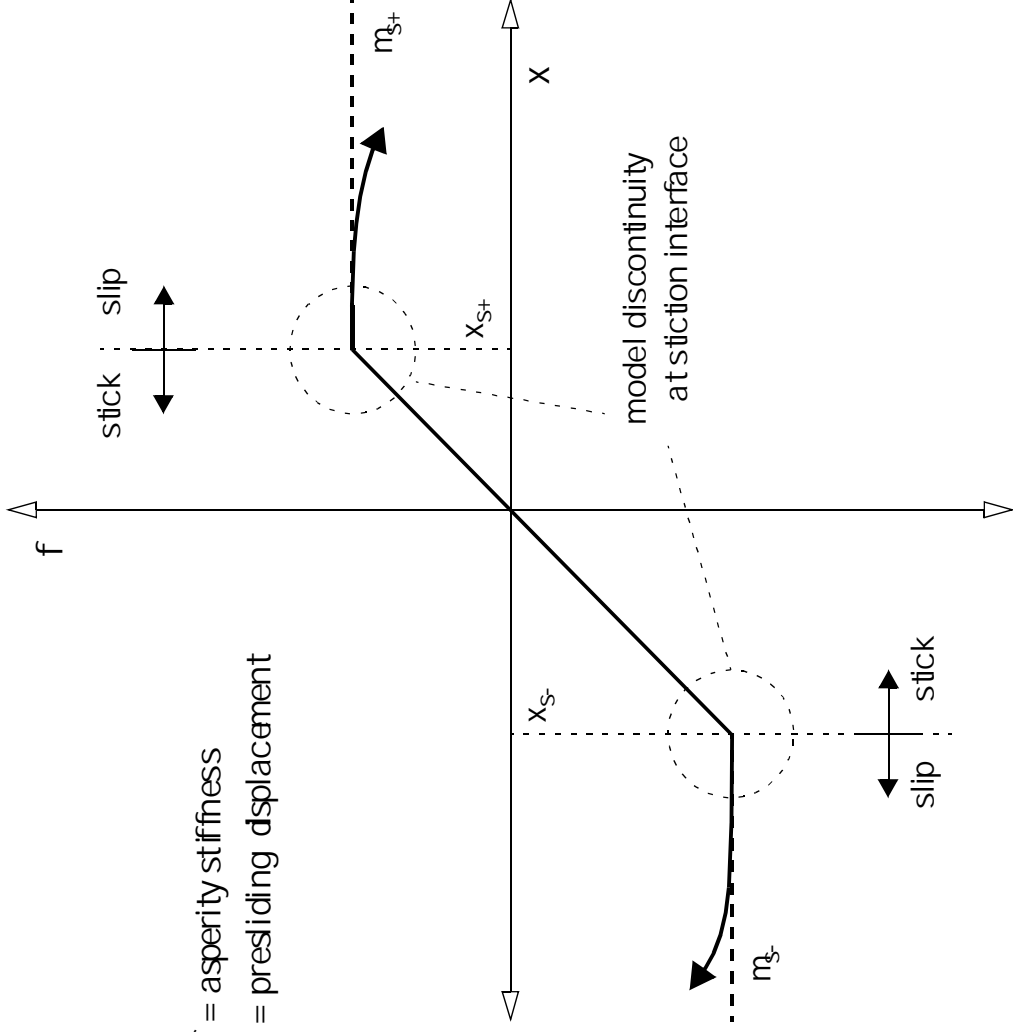
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- Kato (1972), Armstrong-Hélouvy (1990, 1993)
  - time at rest (dwell time) allows the surface asperities to plastically deform due to contact pressure (“rising friction”)
  - lubricated contacts also gradually squeeze out trapped lubricant pockets (“starvation”)

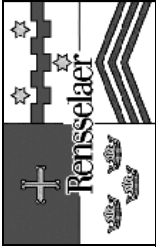
# Presliding Friction

- Armstrong-Hélouvy / Dupont (1993)

$f$  = asperity stiffness  
 $x$  = presliding displacement



- constraint force acting at zero velocity
- tangential elastic deformation of surface asperities
- slope discontinuity at the zero-velocity interface, but functional continuity is preserved
- presliding friction can be used to predict the breakaway static friction force

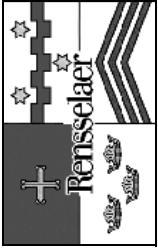


# Stribeck Friction

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- Stribeck (1907)

- continuous transition from static to kinetic friction after breakaway occurs
- decreasing friction force with increasing load, up to a point, and then increasing with load
- decreasing friction force with increasing temperature and longer operation

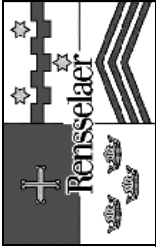


# Normal-Force Friction

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- Stribeck (1907)

- friction at a lubricated interface is modulated by the contact pressure, a function of contact load and stiffness
- friction depends on the lubricity, which increases with temperature (as viscosity decreases)
- kinetic friction dominates over viscous friction under high bearing loads



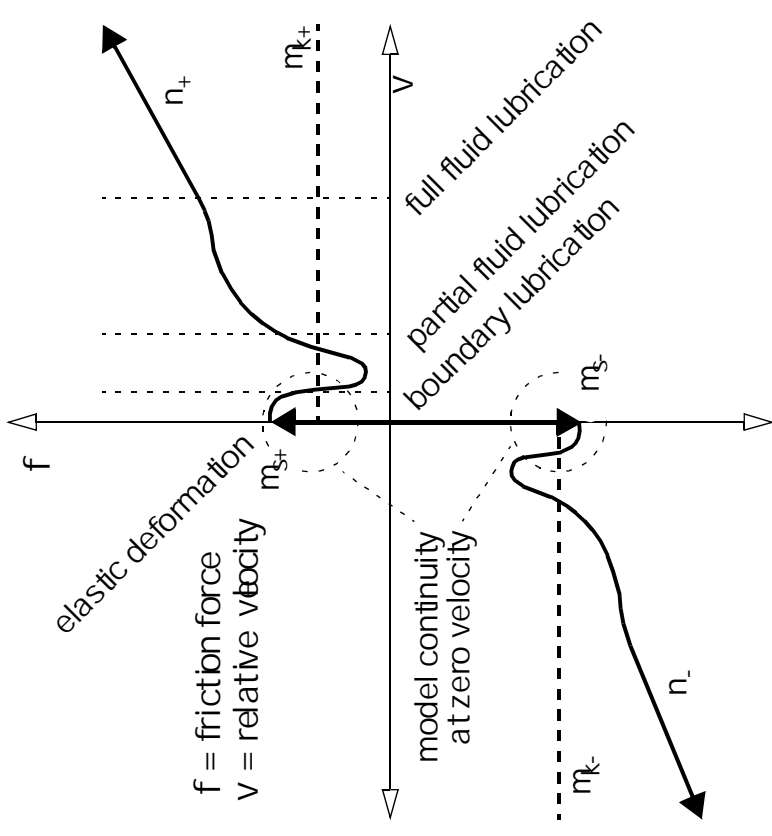
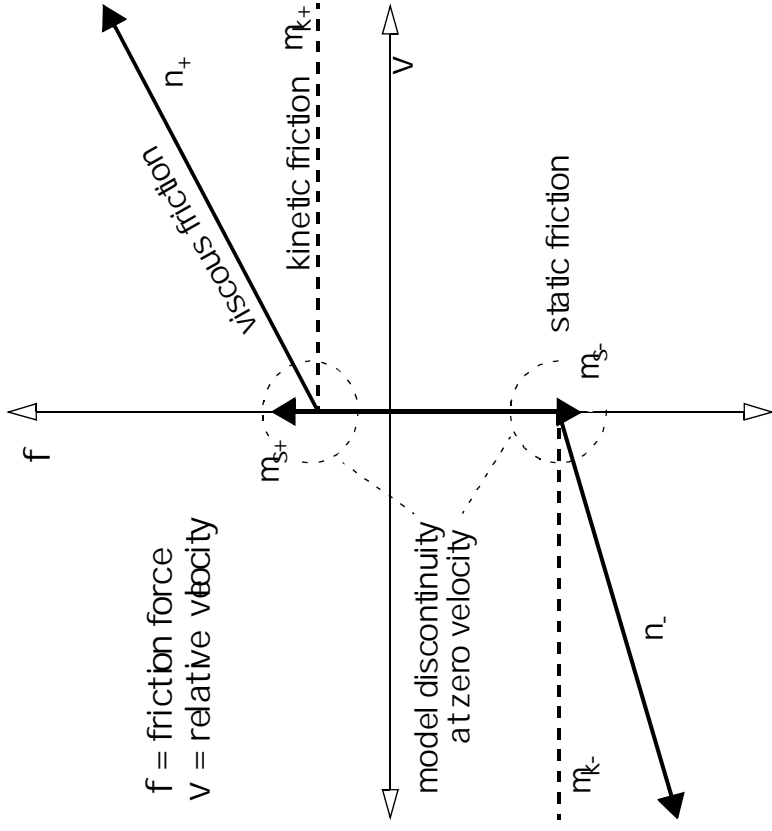
# Frictional Memory

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- Hess/Soom (1990) Armstrong-Hélouvy / Dupont (1993)

- changes in velocity result in a lagging change in friction
- the lag is attributed to shear reversal in the lubricant layer during a velocity reversal

# Dynamic Friction Model



- Traditional model
  - discontinuity from static to kinetic

- Modern model
  - captures the dynamic complexity of stiction and low velocity motion



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# Backlash Model (new contribution)

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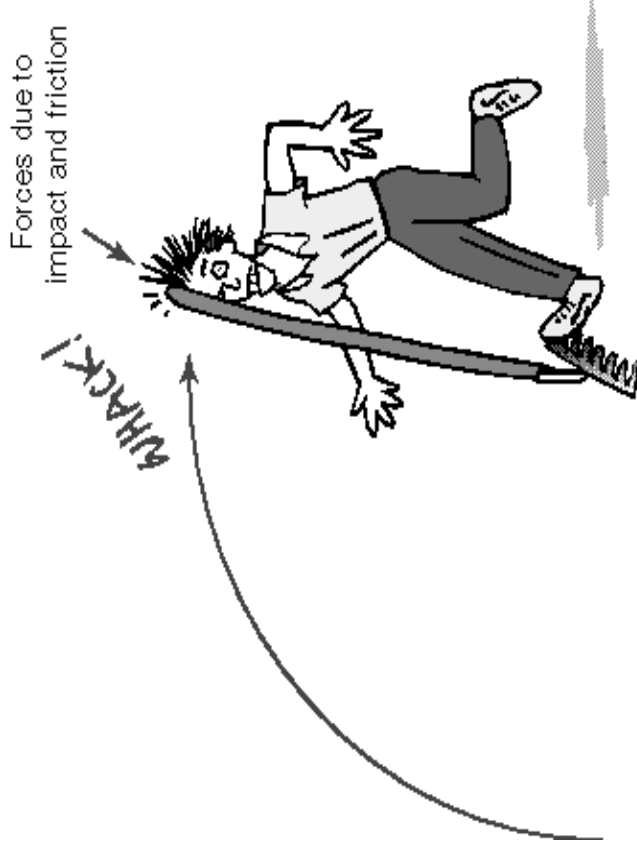
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Slap-stick friction with backlash.



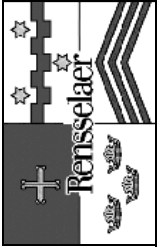
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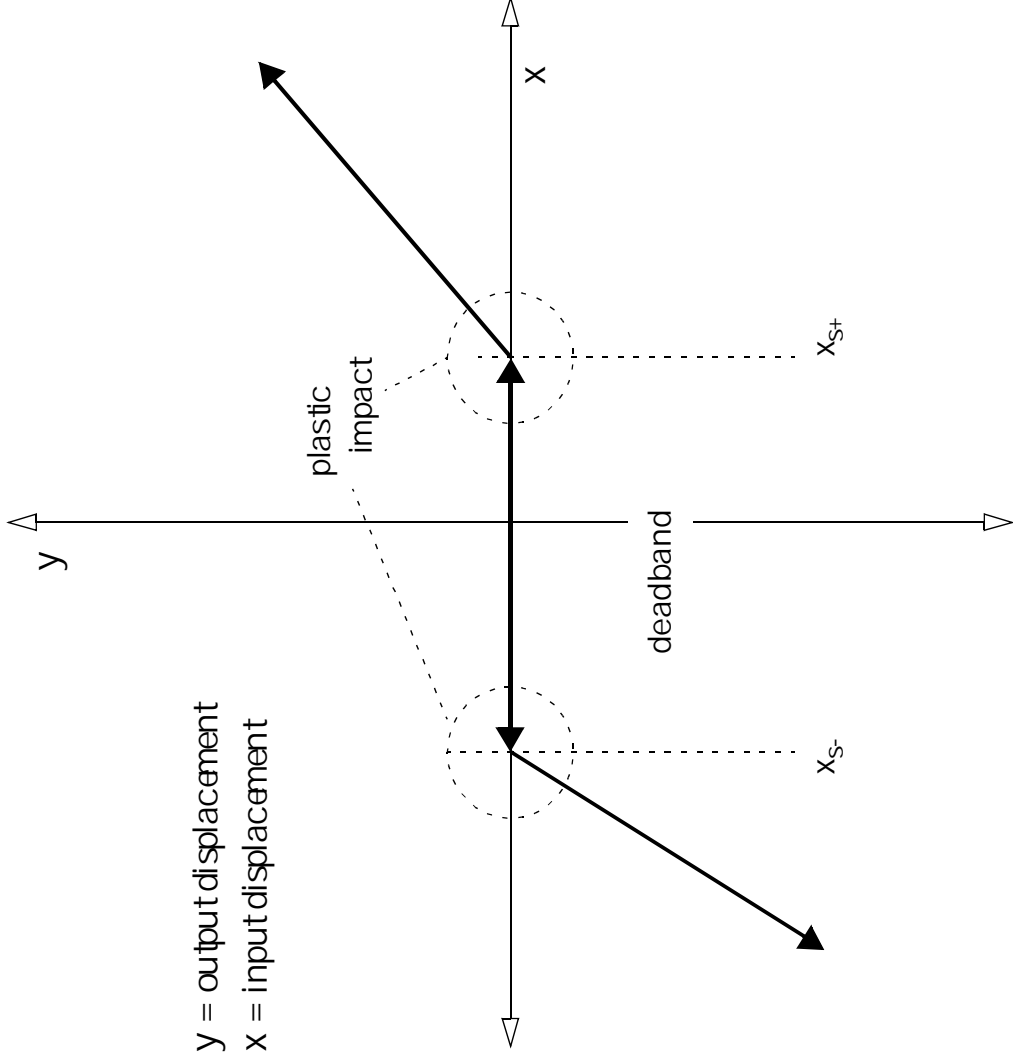


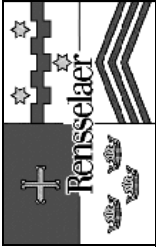
# The Classical Backlash Model

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• Tao / Kokotovic (1994)

- developed in detail for control system design over the past decade
- considers only the phase lag induced by loss of coupling
- implicitly assumes that all rigid-body impacts are fully plastic, and that momentum is conserved
- less detailed than coefficient of restitution model

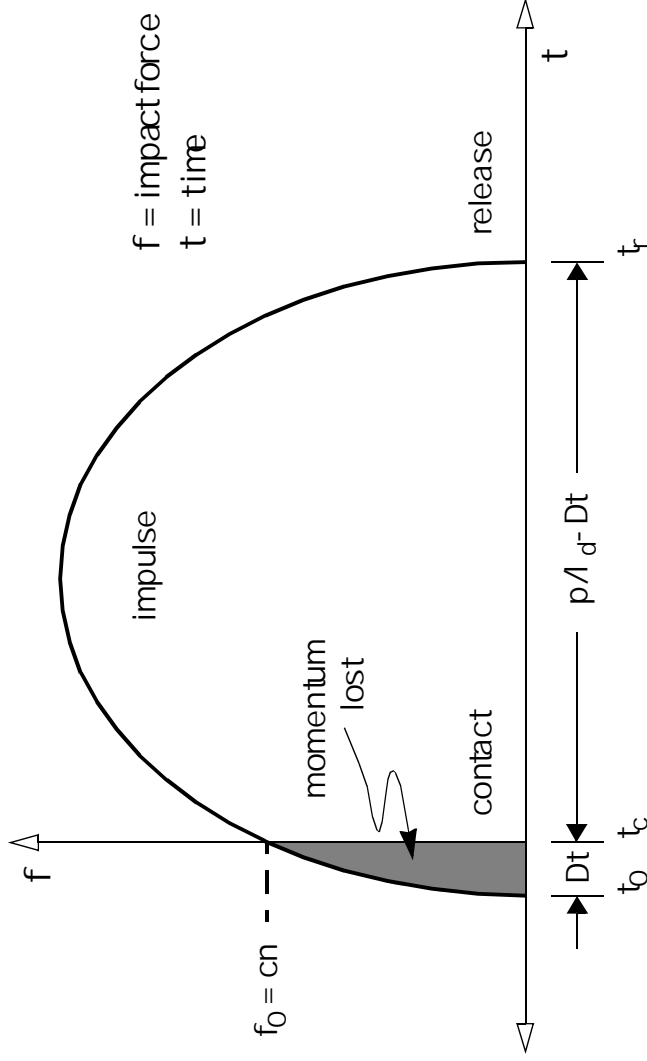


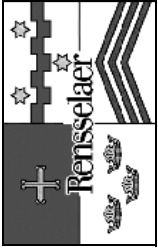


# Viscoelastic Impact Effects

## • Lee/Scarton (1995)

- extension of "Scarton" Dynamic Hardness (SDH) model to include momentum effects as well as viscoelastic velocity reversal
- impulse force delivered over a brief instant
- the integral of the impact force is the momentum "lost" during impact
- model includes the impact force delivered, and the duration of impact, giving a momentum estimation

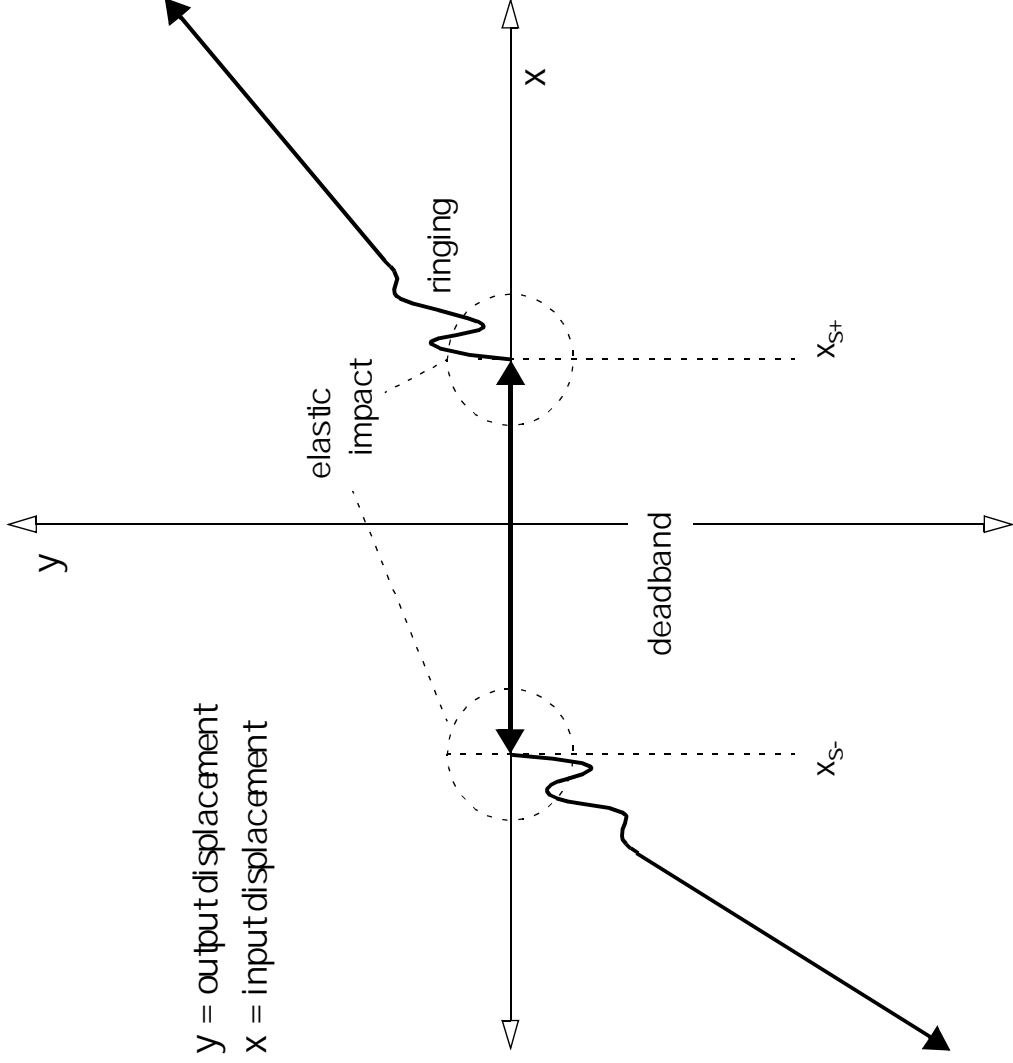




# Backlash with Impact Effects

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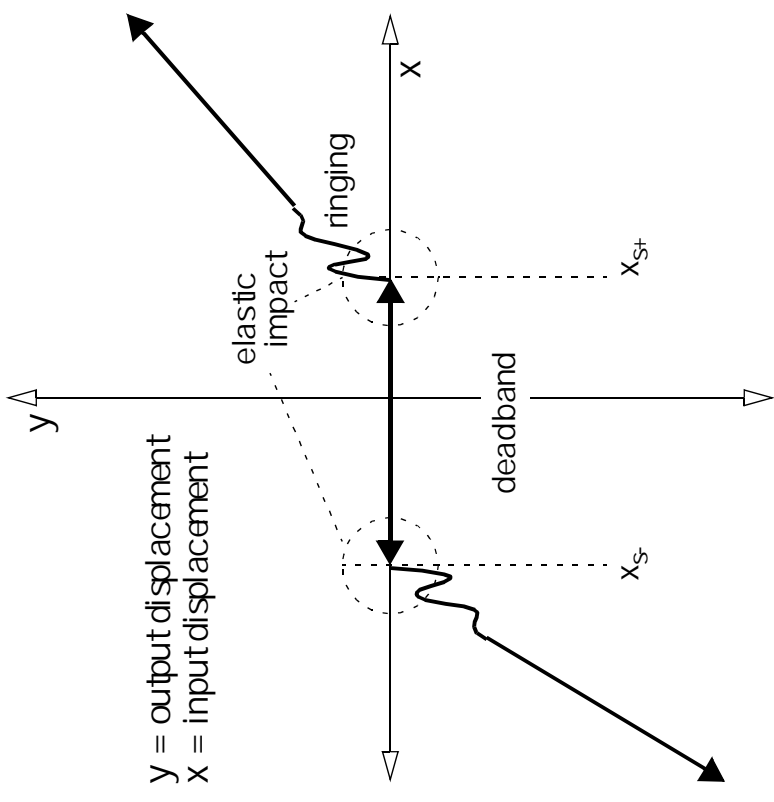
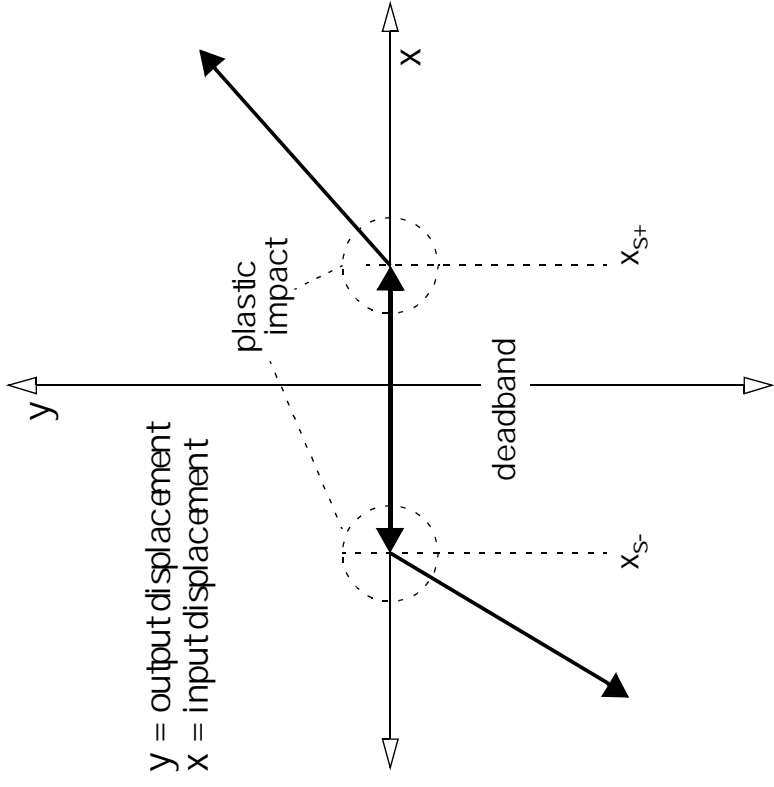
• de Marchi / Craig (1997)



$y$  = output displacement  
 $x$  = input displacement

- with machines operating at higher speeds, forceful impact across a deadzone can cause vibrations, noise, and damage
- when the impacted body's compliant, the vibrational component is significant
- improved controllability requires an account of impact-induced vibrations

# Impact Backlash Model



- Traditional model .....
  - the backlash has no dynamic component

- Modern model (new)
  - impact-induced vibration is fully included in the dynamics



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# Compliance Model

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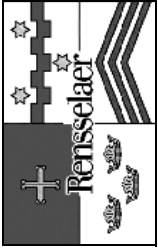
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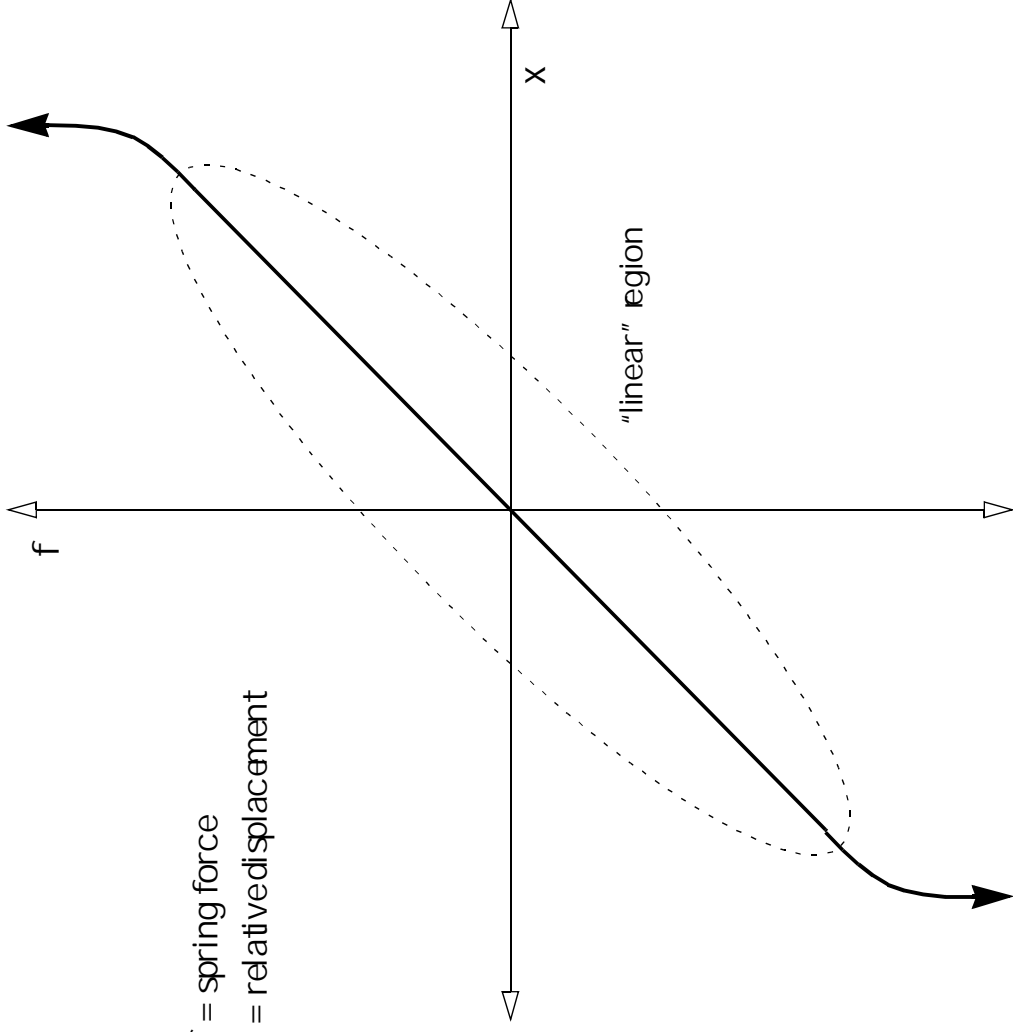
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# Classical Compliance Model

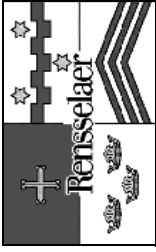
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$f$  = spring force  
 $x$  = relative displacement

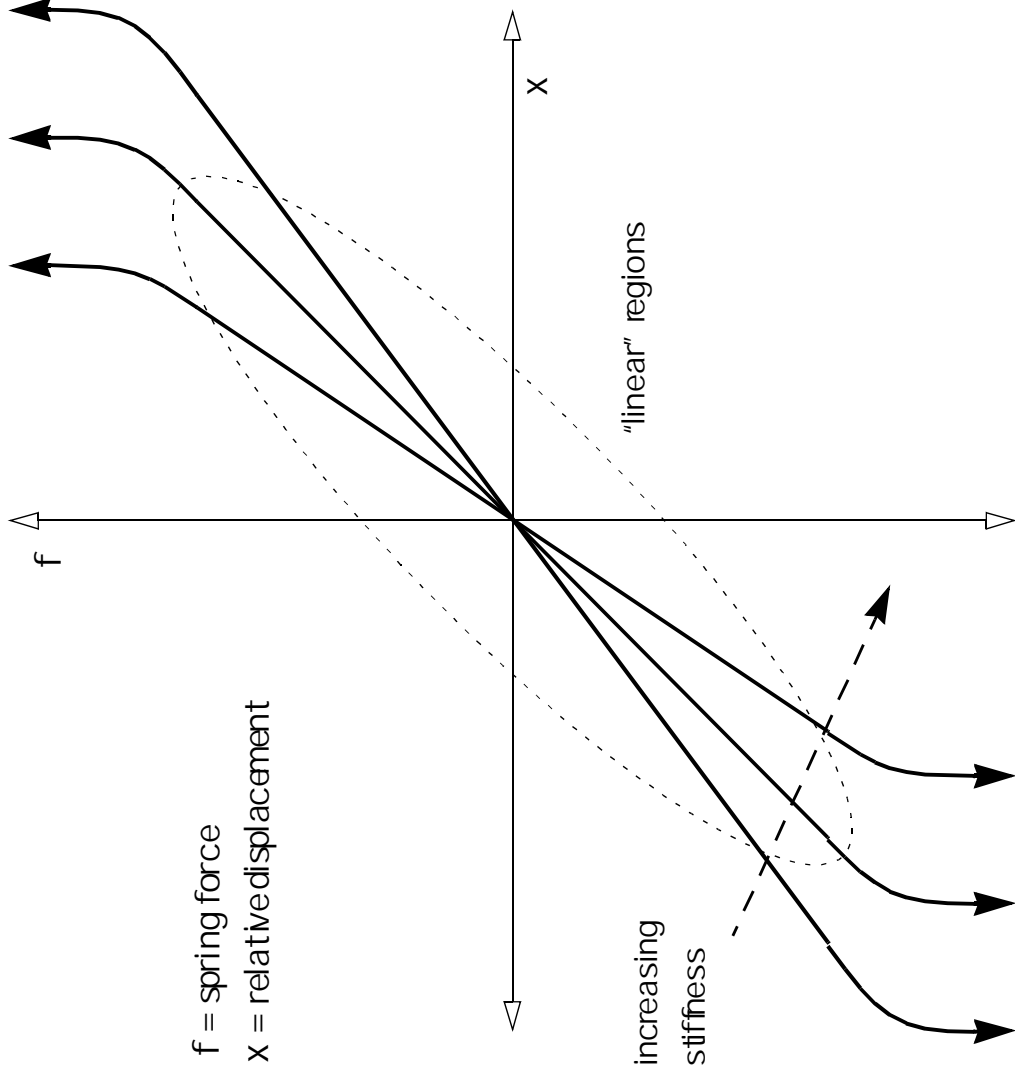
- Hooke (1660)

- describes the proportionality of stress and strain in springs ("Hooke's Law")
- a linear model which is only valid for elastic deformations with limited range
- nonlinearity becomes evident with extreme amplitudes



# Summed-Mode Compliance

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$f$  = spring force

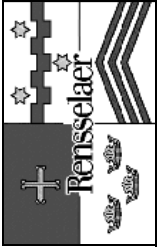
$x$  = relative displacement

increasing  
stiffness

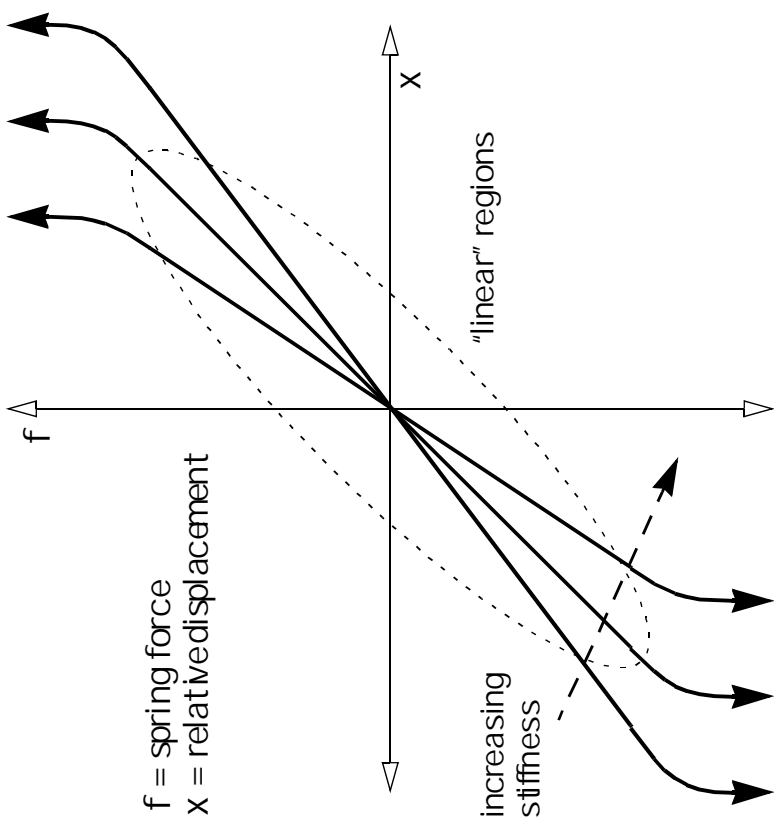
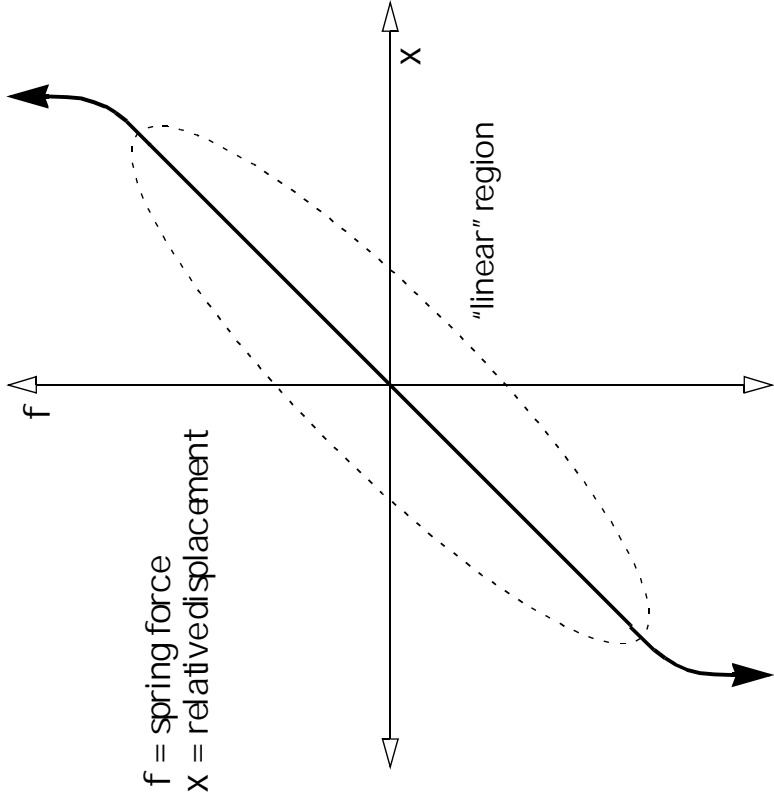
"linear" regions

• Rayleigh (1877), Thomson (1972)

- assumes that damping is proportional to frequency
- many "spring constants" summed over a finite number of structural vibration frequencies
- nonlinear spring force used at high amplitudes
- nonlinear model varies with amplitude and frequency



# Elastic Compliance Model



- Traditional model .....
  - spring force is unimodal and linear

- Modern model
  - multiple modes and nonlinear spring forces are included





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# Lumped Parameter Model and Actual Mechanical System

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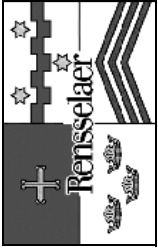
Prof. Dr. Daniel F. Walczyk, Committee Member

Prof. Dr. James Napolitano, Committee Member



Julian A. de Marchi

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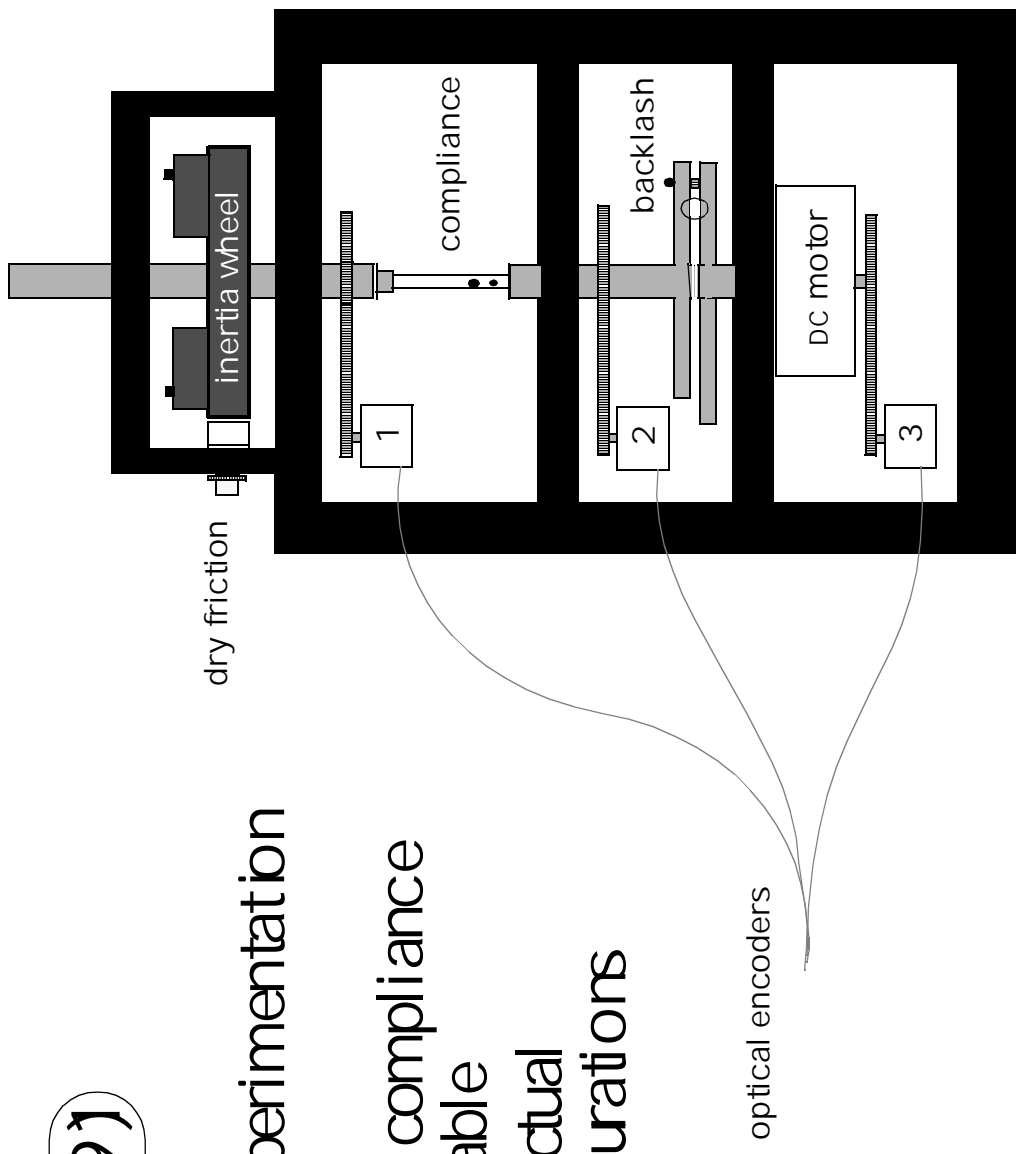


# Mech. Positioning Test Bed

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- Walczyk / Craig (1991)

- unique test bed for experimentation and validation
- friction, backlash and compliance are arbitrarily adjustable
- can mimick various actual machines and configurations

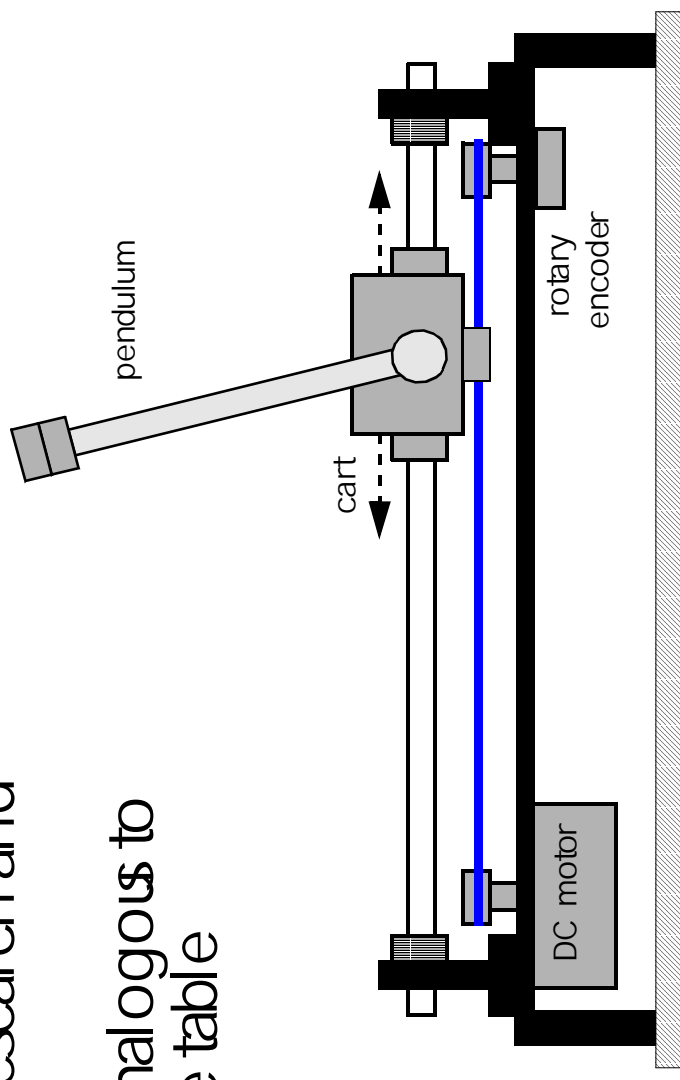


# Linear Inverted Pendulum

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- T üfekçi /de Marchi /Craig (1998)

- mechatronic device for research and education
- linear table movement analogous to machine tool workpiece table





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# System Identification Algorithms (new contributions)

Ph.D. Thesis Defence

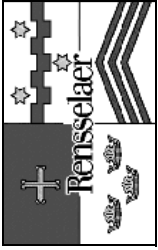
by Julian A. de Marchi, N.S.F. Graduate Fellow

Prof. Dr. Kevin C. Craig, Committee Chair

Prof. Dr. C. James Li, Committee Member

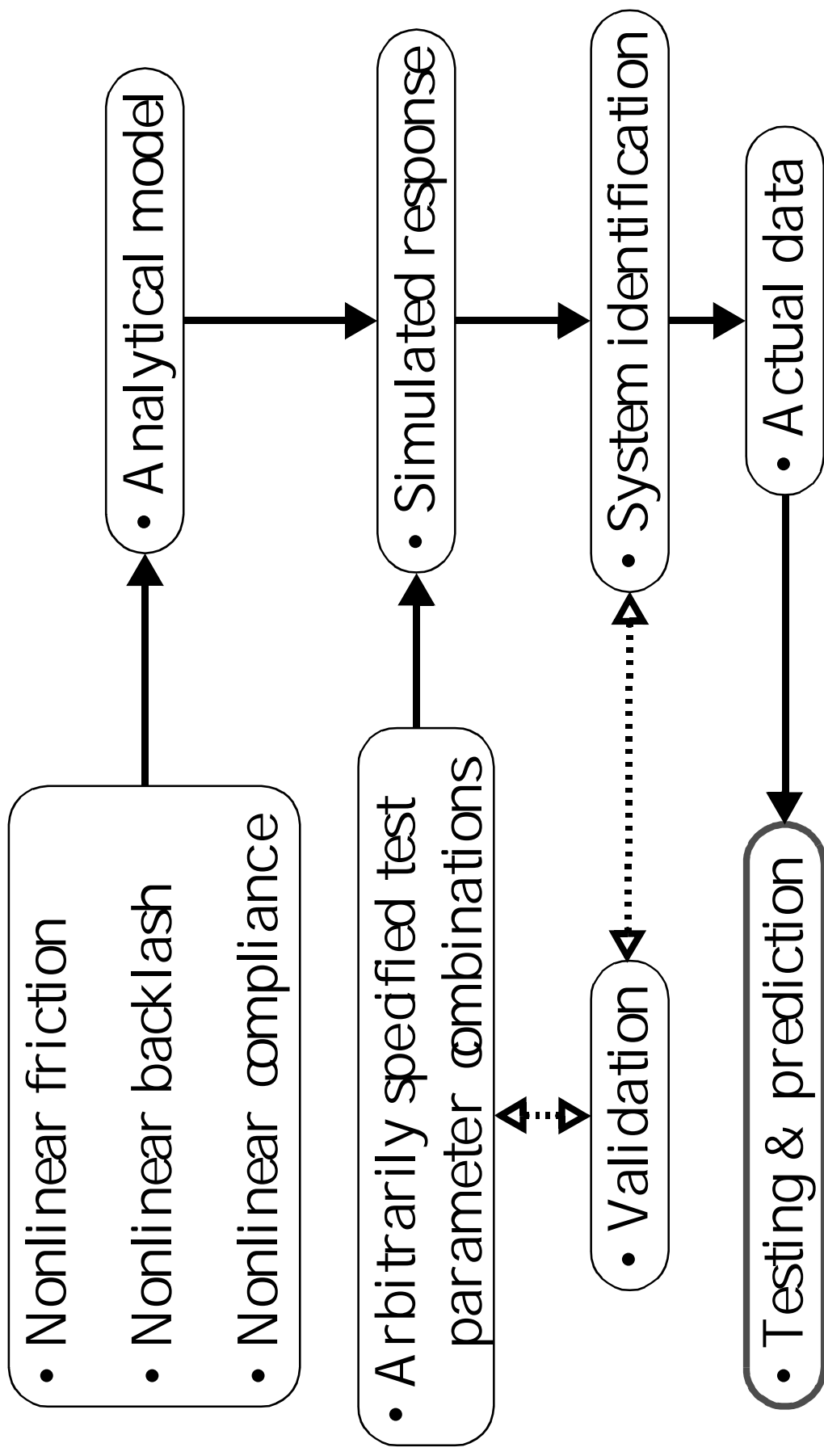
Prof. Dr. Daniel F. Walczyk, Committee Member

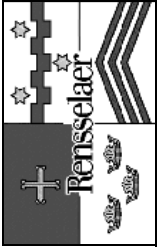
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# The System ID Process

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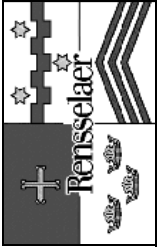


# System ID Methods Used

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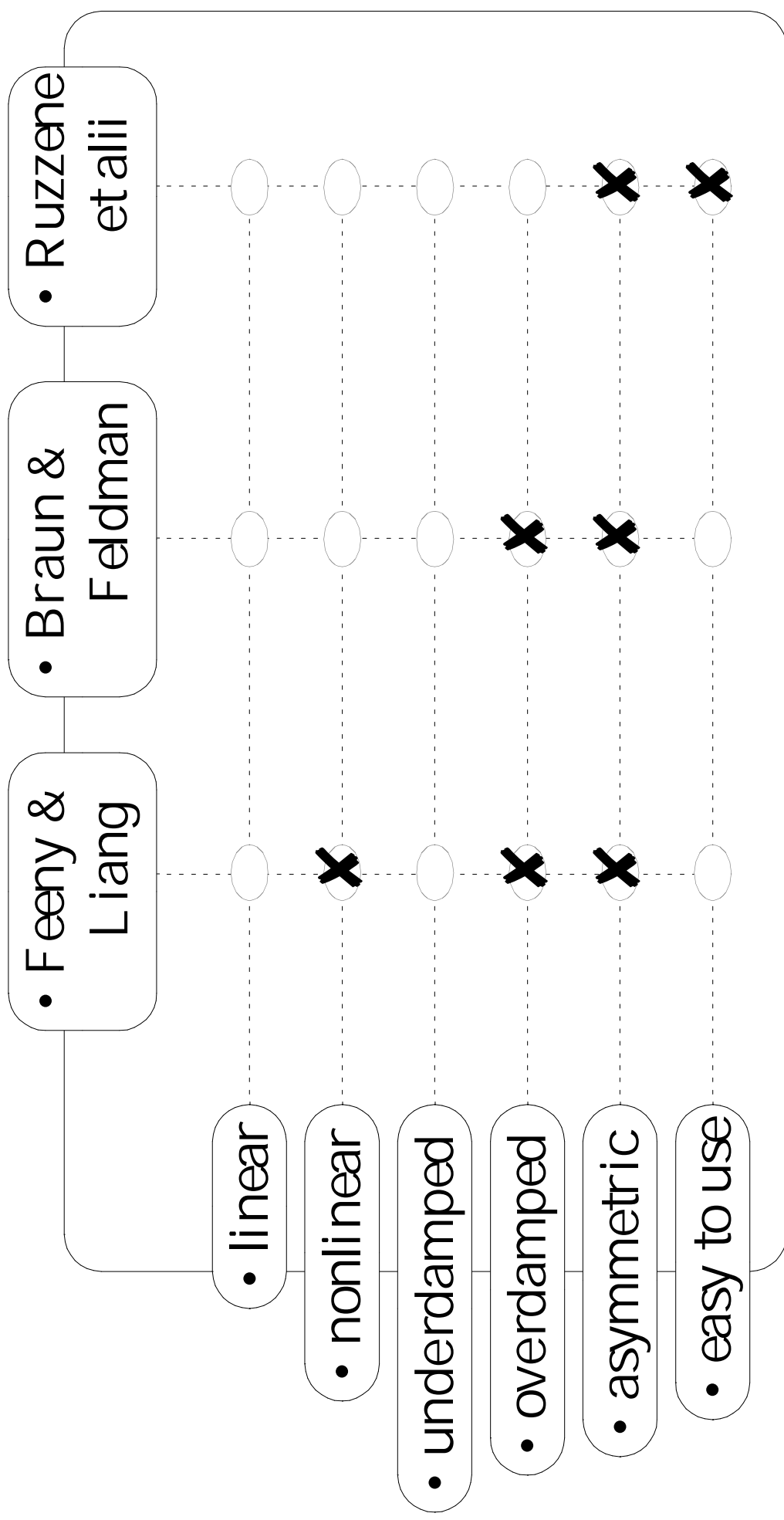
- Pros: Logarithmic decrement method • Cons:
  - static, kinetic, and viscous friction
  - linear compliance
- Pros: Hilbert Transform
  - kinetic and viscous friction
  - unimodal nonlinear compliance
  - backlash with impact
- Cons:
  - requires filtering
  - inaccuracies are exacerbated when damping is high

- Pros: Wavelet Transform
  - multimodal nonlinear compliance
  - cleaner data than Hilbert Transform

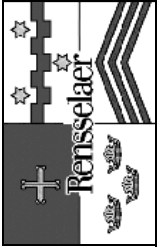


# Existing System ID Limitations

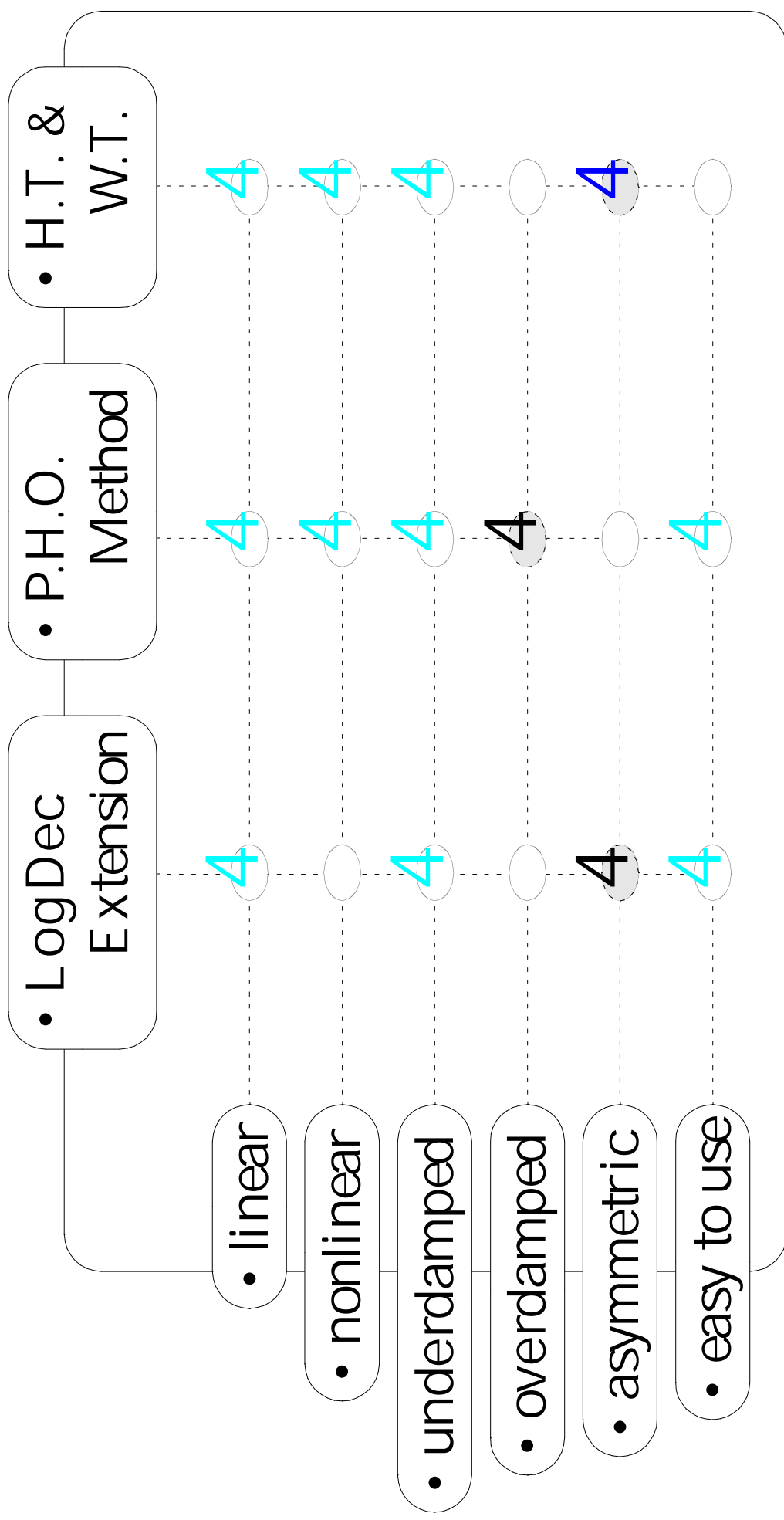
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# New Solutions / Contributions





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# A symmetric Viscous + Kinetic Friction I D: Mathematical Details

Ph.D. Thesis Defence

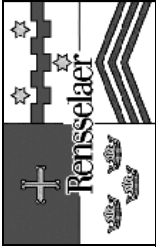
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# Simple Harmonic Motion

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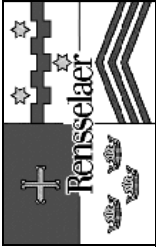
- Simple harmonic motion

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) + f_c = F(t)$$

- Coulomb Friction

$$f_c = \begin{cases} f_s & \dot{x}(t) = 0 \\ f_k \operatorname{sgn} \dot{x} & \dot{x}(t) \neq 0 \end{cases}$$

where  $f_s \neq f_k$



# Kinetic Friction

---

- Static friction stasis

$$x(t) \in x_s \quad \text{where} \quad x_s = f_s/k$$

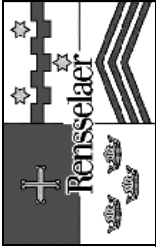
- Kinetic friction stasis

$$x_k = f_k/k$$

- Quasi-linearization

$$m\ddot{y}(t) + c\dot{y}(t) + ky(t) = F(t)$$

where  $y(t) = x(t) + x_k \operatorname{sgn} \dot{x}(t)$  and  $\dot{y}(t) = \dot{x}(t)$



# A symmetric Friction

---

- A symmetric friction

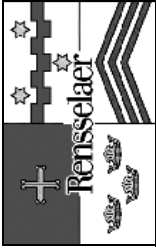
$$x_s = \bar{x}_s + Dx_s \operatorname{sgn} F(t)$$

$$x_k = \bar{x}_k + Dx_k \operatorname{sgn} x(t)$$

$$s = \bar{s} + Ds \operatorname{sgn} \dot{x}(t)$$

- Quasi-linearization

$$y(t) = x(t) + \bar{x}_k + Dx_k \operatorname{sgn} y(t)$$



# Free Harmonic Oscillation

---

- General solution

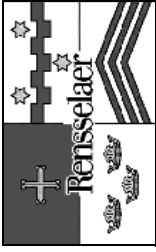
$$y(t) = Ae^{-st} \sinh(\omega_d t - j \phi)$$

where  $\omega_d = \sqrt{s^2 - \omega_0^2}$

- Underdamped solution

$$y(t) = Ae^{-st} \sin(\omega_d t - j \phi)$$

where  $\omega_d = \sqrt{\omega_0^2 - s^2}$



# Log. Decrement Method

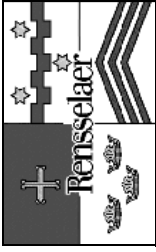
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- Peak displacements

$$y_n = (-1)^n y_0 e^{-npb} = -y_{n-1} e^{-npb}$$

- Logarithmic decrement

$$pb = -\log \frac{y_n}{y_{n-1}} = -\frac{s}{W_d} = -\frac{z}{\sqrt{1-z^2}}$$



# A symmetric Viscous Friction

---

- Peak displacements with asymmetric friction

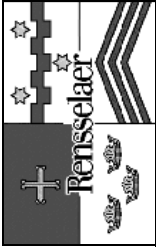
$$X_n + X_{n-1}e^{-pa} = -(1 + e^{-pa})[DX_k + (-1)^n \bar{x}_k]$$

$$X_{n-1} + X_{n-2}e^{-pb} = -(1 + e^{-pb})[DX_k + (-1)^{n-1} \bar{x}_k]$$

- Viscous friction estimation

$$e^{-pa} = -\frac{Y_n - Y_{n-2}}{Y_{n-1} - Y_{n-3}} \quad \text{and} \quad e^{-pb} = -\frac{Y_{n-1} - Y_{n-3}}{Y_{n-2} - Y_{n-4}}$$





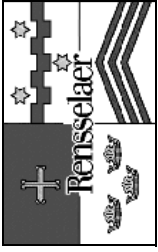
# A symmetric Kinetic Friction

---

- Kinetic friction estimation

$$\bar{x}_k = \frac{1}{2(-1)^n} \frac{y_n + y_{n-1}e^{-pa}}{1 + e^{-pa}} - \frac{y_{n-1} + y_{n-2}e^{-pb}}{1 + e^{-pb}} \dot{f}$$

$$Dx_k = \frac{1}{2} \frac{y_n + y_{n-1}e^{-pa}}{1 + e^{-pa}} + \frac{y_{n-1} + y_{n-2}e^{-pb}}{1 + e^{-pb}} \dot{f}$$



# Forced Harmonic Oscillation

---

- Forced harmonic oscillation

$$F(t) = B \sin(\omega t - \phi)$$

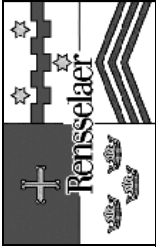
- Steady-state solution

$$y_p(t \gg t_0) = C \sin(\omega t - \phi)$$

- System ID

$$\omega_0^2 = \omega^2 + \frac{R \cos \phi}{m} \quad \text{and} \quad S = \frac{R \sin \phi}{2m\omega}$$

where  $R = B/C$  and  $\phi = \phi - \phi_0$



# Parametric Harmonic Osc

---

- Parametric harmonic oscillation (PHO) via PD feedback

$$m\ddot{y}(t) + c\dot{y}(t) + ky(t) = F(t) = - [D_i\dot{y}(t) + P_i y(t)]$$

$$m\ddot{y}(t) + c_i\dot{y}(t) + k_i y(t) = F(t)$$

where  $c_i = c + D_i$  and  $k_i = k + P_i$

- Physical and dimensionless parameter relationships

$$c_i = 2m\sigma_i \quad \text{and} \quad k_i = m\omega_{oi}^2$$

# Physical Parameter Estimation

---

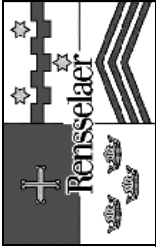
- (Pseudo) Freeparametric oscillation

$$\hat{m} = \frac{D_i - D_j}{2(s_i - s_j)} \quad \hat{c} = \frac{D_i s_j - D_j s_i}{s_i - s_j} \quad \hat{k} = \frac{\hat{m}(w_{0i}^2 + w_{0j}^2)}{2}$$

where  $D_i \neq D_j$

$$\hat{m} = \frac{P_i - P_j}{2(w_{0i}^2 - w_{0j}^2)} \quad \hat{k} = \frac{P_i w_{0j}^2 - P_j w_{0i}^2}{w_{0i}^2 - w_{0j}^2} \quad \hat{c} = \hat{m}(s_i + s_j)$$

where  $P_i \neq P_j$



# Physical Parameter Estimation

---

- (Pseudo) Forced parametric oscillation

$$\hat{m} = \frac{(P_i - R_i \cos r_i) - (P_j - R_j \cos r_j)}{w_i^2 - w_j^2}$$

$$\hat{c} = \frac{(R_i \sin r_i - D_i) w_j + (R_j \sin r_j - D_j) w_i}{2w_i w_j}$$

$$\hat{k} = \frac{(P_i - R_i \cos r_i) w_j^2 - (P_j - R_j \cos r_j) w_i^2}{w_i^2 - w_j^2}$$

(with no restriction on  $P_i$ ,  $D_i$ )



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# Friction Identification: An Example



" Grunt! I estimate of coefficient correct  
we have fire by now! "

Ph.D. Thesis Defence

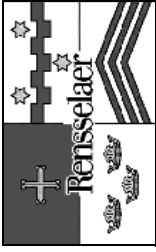
by Julian A. de Marchi, N.S.F. Graduate Fellow

Prof. Dr. Kevin C. Craig, Committee Chair

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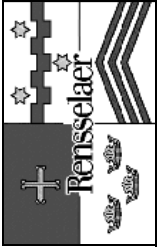
Prof. Dr. James Napolitano, Committee Member



# Friction I D Example

---

- System without backlash
  - Coulomb friction
  - viscous friction
  - frictional bias
  - compliance
- Excited via the parametric harmonic oscillation method
  - free vibration used as input to logarithmic decrement, Hilbert and wavelet analyses
  - dual analyses yield redundancy
  - also provide different strengths

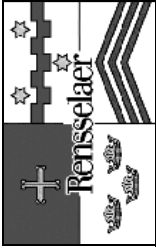


# Experimental Data

---

- Log decrement method
  - estimates compliance, Coulomb friction and bias
- Hilbert Transform method
  - estimates friction and compliance; also can estimate backlash



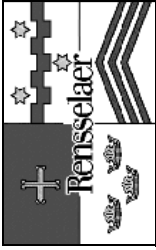


# Dataset #1

---

	Dataset #1	
	measured	estimated
$P_i = 0.6212$ ( $\text{Nms}^2$ )		
$w_0$ (rad/s)	7.4337	7.5822
$\bar{z}$	0.0160	0.0162
Dz	0.0028	0.0028
$\bar{x}_k$ (rad)	0.0780	0.0780
$Dx_k$ (rad)	0.0351	0.0351

- Note sensitivity to the viscous friction estimation



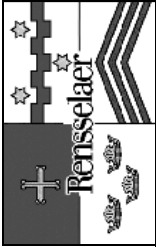
# Dataset #2

---

$P_i = 0.5590$   
( $\text{Nms}^2$ )

	Dataset #2	
	measured	estimated
$w_0$ (rad/s)	7.0008	7.0951
$\bar{z}$	0.0147	.0148
Dz	0.0086	.0036
$\bar{x}_k$ (rad)	0.0792	.0792
$Dx_k$ (rad)	0.0330	.0331

- Note sensitivity to the natural frequency estimation



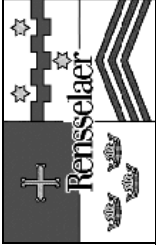
# Dataset #3

---

$P_i = 0.4969$   
( $\text{Nms}^2$ )

	Dataset #3	
	measured	estimated
$w_0$ (rad/s)	6.7371	6.7338
$\bar{z}$	0.0124	0.0121
Dz	0.0026	0.0026
$\bar{x}_k$ (rad)	0.0731	0.0730
$Dx_k$ (rad)	0.0381	0.0381

- Note again the viscous friction sensitivity



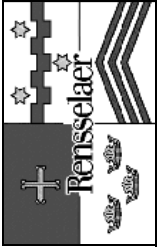
# Dataset #4

---

$P_i = 0.4348$   
( $\text{Nms}^2$ )

	Dataset #4	
	measured	estimated
$\omega_0$ (rad/s)	6.1495	6.1289
$\bar{z}$	0.009	0.0077
Dz	0.0067	0.0067
$\bar{x}_k$ (rad)	0.0660	0.0660
$Dx_k$ (rad)	0.0380	0.0381

- Note again the natural frequency sensitivity.



# Physical Parameter ID

---

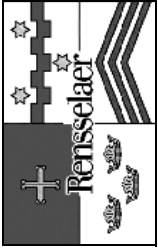
- Massparameter estimation:

$$\hat{m} = \frac{(P_i - R_i \cos r_i) - (P_j - R_j \cos r_j)}{w_i^2 - w_j^2}$$

$$\hat{c} = 2\hat{m}s_j = 2\hat{m}z_j w_j \quad \hat{k} = \hat{m}w_0^2$$

- $\hat{m} \gg 5.58 \cdot 10^{-4} \text{Nms}$
- $m = 6.09 \cdot 10^{-4} \text{Nms}$
- $f_k \gg 0.04 - 0.02 \text{Nm}$

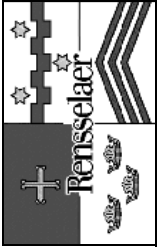
	P.H.O. #1	P.H.O. #2	P.H.O. #3	P.H.O. #4	F.H.O.
$P_i$ (Nms <sup>2</sup> )	0.6212	0.5590	0.4969	0.4348	(0)
$\bar{c}_i$ (Nms)	0.0028	0.0024	0.0019	0.0011	-0.0025
$Dc_i$ (Nms)	0.0004920	0.0005853	0.0004068	0.0009568	N/A
$k_i$ (Nm/rad)	0.6416	0.5691	0.5270	0.4391	-0.0079



Julian A. de Marchi

# Log. Decrement Simulation

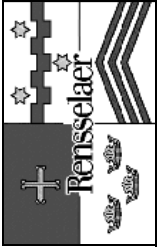
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# Actual Pendulum System I D

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# Actual Test Bed System ID

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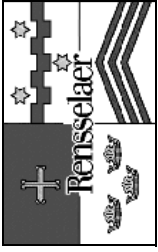
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# Conclusion

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Prof. Dr. Kevin C. Craig, Committee Chair  
Prof. Dr. C. James Li, Committee Member  
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# Continuing Work

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- Identification

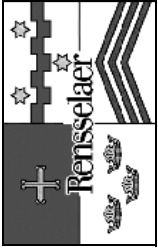
- backlash
- backlash with compliance
- periodic friction
- static and hysteretic friction

- Analysis

- extension of asymmetry to Hilbert & Wavelet Transform analyses

- Application

- automation of identification procedure



# Future Work

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- Application

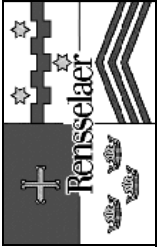
- apply testbed results to an actual machine tool
- motorised workpiece table on drill press in laboratory

- Extension

- on-line and adaptive system ID techniques

- Development

- feedforward control and adaptive control



# Demonstration

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- Nonlinear system modeling and simulation
- Signal processing details
- Mechatronic implementation
- Friction identification
- Backlash identification
- Compliance identification
- Machine tool instrumentation approach



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# Recent Presentations and Articles for Submission

Ph.D. Thesis Defence

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# Supplementary Information

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