MECHATRONICS DESIGN OF AN INVERTED PENDULUM SYSTEM FOR ENGINEERING EDUCATION *

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ABSTRACT

This paper details the mechatronic design process as applied to the design of an inverted pendulum system, for educational use by undergraduate mechatronics students. Conception and development of the design are outlined with an emphasis on mechatronic design principles. Realisation of the design is made with simultaneous consideration towards constraints such as cost, reuse of existing materials, functionality, extensibility and educational merit. It represents a need for sensitivity towards these aforementioned issues, and also, how well the system represents the idealised dynamics presented in the classroom. The system design consists of "upgrading" an existing system, by replacing analogue sensors with digital ones, refitting some rotary bearings, and improving other design features wherever possible and appropriate. The resulting design is a linear-track inverted pendulum system with swing-arround capability, designed to familiarize students with control system design principles.

KEYWORDS: mechatronics, dynamic system investigation, synergism, integration, control system design, mechatronics design, modeling.

1. INTRODUCTION

Mechatronics is the *synergistic* combination of mechanical engineering, electronics, control systems and computers. The key element in mechatronics is the *integration* of these areas through the design process. The concept of synergism and integration through the design process is illustrated in Figure 1. The key to success in mechatronics system design is a balance between two skills:

- modeling (physical and mathematical), analysis (closed-form and numerical solution), and control system design (analogue and digital) of dynamic physical systems
- experimental validation of models and analysis (for computer simulation without experimental verification is at best questionable, and at worst useless) and understanding the key issues in hardware implementation of design

In order to design and build quality precision consumer products in a timely manner, the present-day mechanical engineer must be knowledgeable (both analytically and practically) in many different areas as indicated in Figure 1. The ability to design and implement analogue and digital control systems, with their associated analogue and digital sensors, actuators, and electronics, is an essential skill of every mechanical engineer, as almost everything today needs controls.

*This work is partially funded by Xerox, Inc.

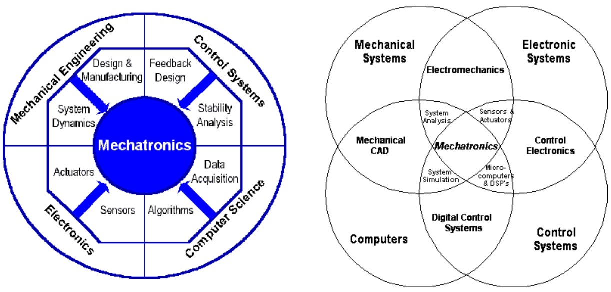

Figure 1. Mechatronics: Synergism and integration through design

Control design is not just for specialists anymore.

Figure 2 shows a diagram of the procedure for *dynamic system investigation* which emphasizes the balance between the aforementioned skills. The dynamic system investigation is a guide for the study of various mechatronic hardware systems. At Rensselaer, students perform a complete dynamic system investigation of mechatronic systems while they develop skills that will be indispensable as mechatronics design engineers in future years [2].

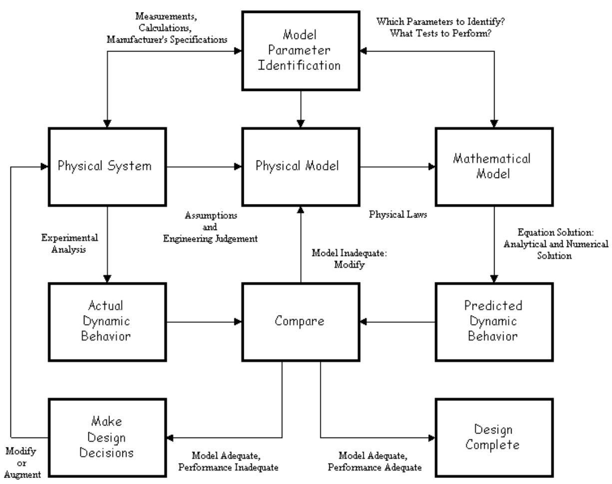

Figure 2. Dynamic system investigation

This paper focuses on the mechatronics design of an inverted pendulum system for engineering education. The presentation here evolves as an educational approach and reflects the coursework offered at Rensselaer.

The inverted pendulum system is a very popular system for controls education [1]. This system is chosen for mechatronic design since it covers almost every area of interest depicted in Figure 1. Some of the other systems used in mechatronics education at Rensselaer are the DC motor, stepper motor, thermal system, pneumatic system, and magnetic levitation system, to which a very similar design procedure is applied. However, it is worthed to mention that every system has its own characteristics.

The use of the dynamic system investigation procedure depicted in Figure 2 for the inverted pendulum system is discussed throughout the paper. The modeling of the inverted pendulum is explored in Section 2. Section 3 explains the details of the inverted pendulum system design for both electrical and mechanical aspects. Control system design and implementation is studied in Section 4 and identification for friction is discussed in Section 5.

2. SYSTEM MODELING

The real *physical system* is an inverted pendulum system which can swing around a pivot point attached on a cart. A force is applied to the cart to keep the cart position and the pendulum angular position, at the desired values, i.e. the cart centered and the stick balanced upwards. In order to keep this inherently unstable system under control, the required force is applied by an electric motor and the feedback is provided by optical encoders.

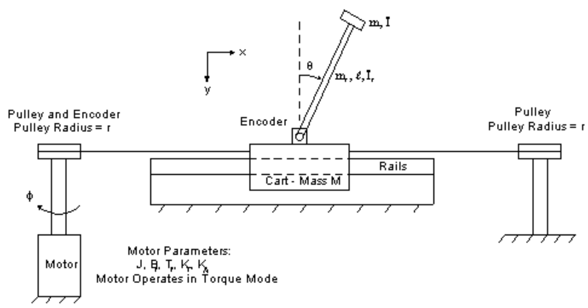

Figure 3. Inverted pendulum physical model

The *physical model* for the system is constructed using lumped physical elements, i.e., rigid rod, lumped mass, etc. After applying a set of simplifying assumptions along with use of engineering judgement, the physical model shown in Figure 3 results.

The *mathematical model* of the plant may be obtained by using several different methods, i.e., D'Alembert's, Newton's, Lagrange's, Kane's etc. One of the above methods is applied after drawing the free-body-diagrams depicted in Figure 4 and the equations (1) and (2) of motion are found which describe the non-linear plant dynamics.

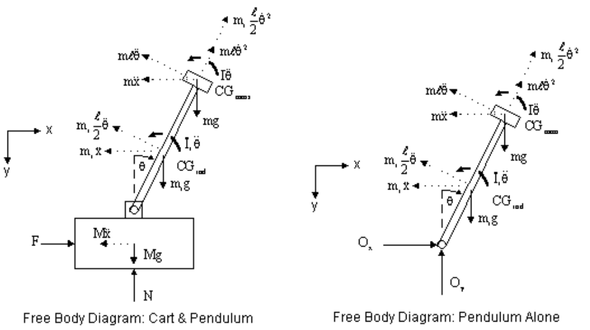

Figure 4. Inverted pendulum free-body-diagrams

$$\begin{aligned} F &= (M + m_r + m)\ddot{x} \\ &\quad + (m + \frac{m_r}{2})\ell\ddot{\theta}\cos\theta \\ &\quad - (m + \frac{m_r}{2})\ell\dot{\theta}^2\sin\theta \qquad (1) \end{aligned}$$

$$\begin{aligned} 0 &= (\bar{I}_r + \bar{I} + m_r\frac{\ell^2}{4} + m\ell^2)\ddot{\theta} \\ &\quad + (m + \frac{m_r}{2})\ell\ddot{x}\cos\theta \\ &\quad - (m + \frac{m_r}{2})g\ell\sin\theta \qquad (2) \end{aligned}$$

We can now find linearised equations of motion about the operating points of $\bar{x} = 0$, $\bar{\theta} = 0$ in equations (3) and (4) by use of Taylor's series expansion.

The force appeared in equation (1) can be calculated using a torque generated by the electric motor. The dynamic relation for this transformation is given in equation (5) which may be substituted into equation (3) to obtain the relation between outputs, x, the cart position, θ, the pendulum angle, and inputs, V_{in}, the input voltage, T_f, the friction torque.

$$\begin{aligned} F &= (M + m_r + m)\ddot{x} \\ &\quad + (m + \frac{m_r}{2})\ell\ddot{\theta} \qquad (3) \end{aligned}$$

$$\begin{aligned} 0 &= (\bar{I}_r + \bar{I} + m_r\frac{\ell^2}{4} + m\ell^2)\ddot{\theta} \\ &\quad + (m + \frac{m_r}{2})\ell\ddot{x} \\ &\quad - (m + \frac{m_r}{2})g\ell\theta \qquad (4) \end{aligned}$$

$$\begin{aligned} F &= \frac{K_t K_a V_{in}}{r} - \frac{J\ddot{x}}{r^2} \\ &\quad - \frac{B_f\dot{x}}{r^2} - \frac{T_f[\mathrm{sign}(\dot{x})]}{r} \qquad (5) \end{aligned}$$

The description of model parameters along with numerical values for the inverted pendulum system is given in Table 1.

3. HARDWARE IMPLEMENTATION

The realisation of the design is made with simultaneous consideration towards constraints such as cost, reuse of existing materials, functionality, extensibility, and educational merit. The inverted pendulum system is inherited from previous years and upgraded to its current state by taking above issues into account. The analogue potentiometers to measure positions are replaced with optical encoders, which are noise-free. The linear track friction is critically reduced and the transmission resolution is improved by replacing a smaller diameter drive wheel. Swing-arround capability is added to the inverted pendulum system to allow design a swing-up controller for self erection. The necessary parts are machined or purchased within a budget, or reused where possible, and assembled to the design specifications.

The inverted pendulum system interface is designed with consideration to the mechanical design. The interface includes selection of hardware actuators, sensors and signal processing, software for interpretation of the sensor signals, and their synthesis to form a closed-loop control signal. The inverted pendulum system and its peripherals form a compound interface depicted in Figure 5.

Table 1
Inverted pendulum parameters

Symbol	Parameter description	Value	Unit
g	gravitational acceleration	9.81	m/s
M	mass of the cart	0.965	kg
ℓ	length of the pendulum	0.636	m
m_r	mass of the pendulum	0.0576	kg
m	mass of the end weight	0.116	kg
$\bar{I}_r$	moment of inertia of pendulum	0.0019	kgm^2
$\bar{I}$	moment of inertia of end mass	1.3127×10^{-5}	kgm^2
J	motor inertia	1.8367×10^{-4}	kgm^2
K_a	servo amplifier gain	1.6	A/V
K_t	motor torque constant	0.1013	Nm/A
B_f	motor viscous friction	1.33×10^{-4}	Nms/rad
r	radius of the pulley wheel	0.01135	m

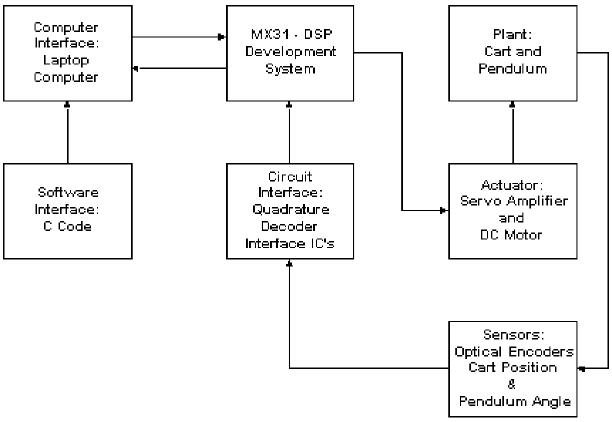

Figure 5. Inverted pendulum and peripherals interface

The transducer placement, precision, and range are integral to both the mechanical and electronics design. Calculations are made to specify the sensing and actuating requirements for the system, and mechanical and electrical components are selected which will work together in a complete design before anything is actually built. The overall system is identified to verify the idealisability of its dynamic behavior.

The complete mechatronic system design thus includes design concept generation, mechanical dynamics analysis, dynamics simulation, component selection and fabrication, electronic hardware and transducer selection and interfacing, circuit design and wiring, software design, system dynamics identification and model verification, and finally controller design and implementation. Each of these steps are highlighted within the scope of the inverted pendulum system, and how mechatronic design principles are key to achieving success on the *first* iteration of the design process.

4. CONTROL SYSTEM DESIGN AND IMPLEMENTATION

The control system architectures are developed for the generic system, and sample controllers are designed for the nominal plant parameters given in Table 1. This is done concurrently with the system development, and once the system identification is performed, actual parameter values may be used to refine the controllers.

The basic approach in the control system design for the inverted pendulum system is to first design the controller in a conventional manner and then discritise it so that the resulting controller can be coded on the computer easily. The C programming language and compilers are used to construct and compile the source code to generate machine code. Then, the object code is dowloaded to an embedded DSP which is the controller running at $f_s = 500\ Hz$.

The control objective for the inverted pendulum system is to keep the pendulum upright in the presence of gravity and unexpected force disturbances applied to the cart or to the pendulum, and to keep the cart centered on the track. In other words, given any initial conditions caused by disturbances, the pendulum can be brought back to the vertical position and also the cart can be brought back to the reference position quickly (e.g., settling time, $t_s \approx 2\ sec$) with reasonable damping (e.g., $\zeta \approx 0.5$).

The classical, state-space, and fuzzy logic controllers are designed in SIMULINK^{R} and implemented on the actual inverted pendulum system. The flowchart shown in Figure 6 explains the general control algorithm implementation in the C programming language.

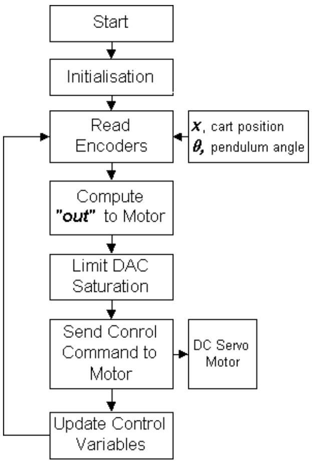

Figure 6. Control algorithm

In the classical control design, first a continuous controller designed for the stick balance, which is called the inner-loop controller as seen in Figure 7. Then, the outer-loop controller designed to bring the cart to the center of the track. Root-locus design techniques are used in the design process and the controllers given in equations (6) are implemented on the DSP.

$$
\begin{aligned}
G_1(s) &= +0.6\frac{s+1}{s+15} \\
G_2(s) &= -23.5\frac{(s+1.5)(s+3)}{s(s+20)} \qquad (6)
\end{aligned}
$$

The controllers given in equation (6) are discritised at a sampling rate of $T_s = 0.002\ s$ using Tustin's method (bilinear transformation [3]) and the resulting difference equations are implemented in the C source code.

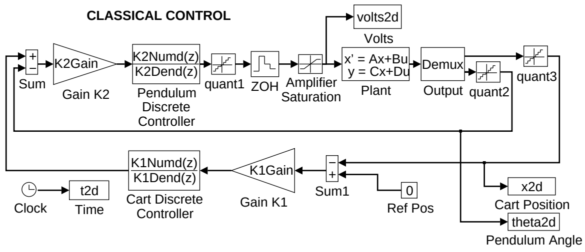

Figure 7. Classical controller

The state-space controller is designed using the linear quadratic state feedback regulator (LQR) method. The linear velocity for the cart and the angular velocity for the pendulum are estimated using a finite difference approximation. The LQR control system simulation is shown in Figure 8 and the following gains given in equation (7) are obtained.

$$
K = [-3.1623\ \ -3.2488\ \ -15.8530\ \ -3.9252] \quad (7)
$$

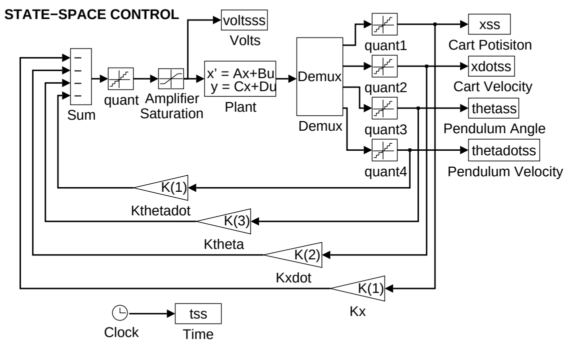

Figure 8. State-space controller

The fuzzy logic controller is designed using the *Fuzzy Logic Toolbox* for MATLAB^{R}. There are 4 pair of membership functions associated with

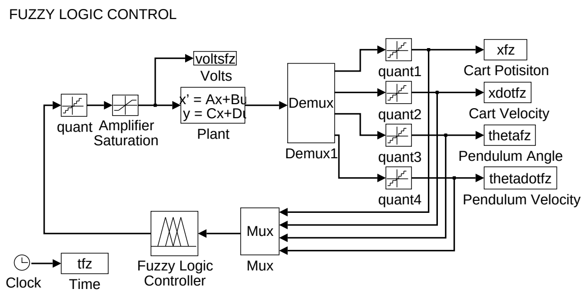

Figure 9. Fuzzy logic controller

each state of the inverted pendulum system. The simulation block diagram is given in Figure 9.

The simulation results are presented in Figure 10 for each of the above three simulations. The experimental results obtained from the state-space controller implementation are shown in Figure 11. We add the dry friction compensation to avoid the dynamic lags and the stick-slip phenomenon. The first of the three plots in Figures 10 and 11 represents the control effort, V_{in} or **out** in *volts*, the second shows the cart position, x in *metres*, and the third depicts the pendulum angle, θ in *radians*. The reason that the experimental results shown in Figure 11 have so many jitter is due to the friction between the cart and the linear track.

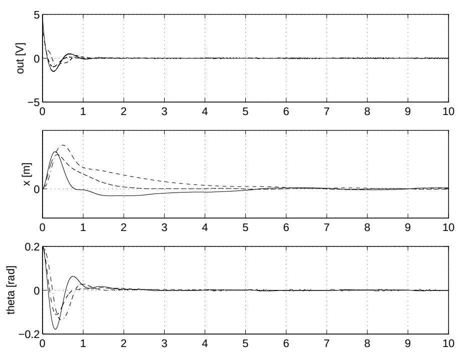

Figure 10. Simulation results (– classical, - - LQR, -.- fuzzy)

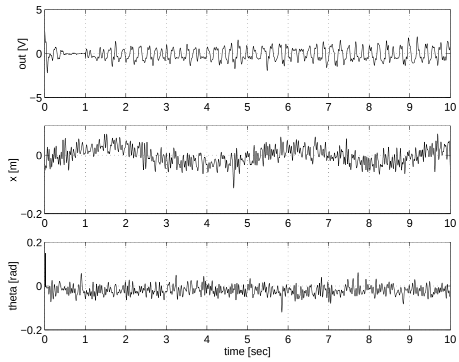

Figure 11. Experimental results for LQR

5. FRICTION IDENTIFICATION

The idealised inverted pendulum system does not exhibit any friction characteristics in the sense that we want the student to conceptually understand the dynamic behaviour of the system and to model and control that behaviour. Including frictional effects complicates the dynamic analysis. Wet (viscous) friction is the only kind which can be linearly modeled, however in the actual system the overall friction acts predominantly like dry (Coulomb) friction, since the bearing lubricant is only a very thin layer of very thin oil. The controller design would therefore require the analysis of a nonlinear system, which is beyond the scope of the coursework. However, the fact that the actual system exhibits significant frictional effects demands that the issue be addressed.

The simplest possible friction model was therefore adopted, which is the traditional textbook model of wet+dry friction as shown in Figure 12.

Because the friction is piecewise continuous, the system is linear as long as the sign of the velocity does not change. This means an analytical solution for the dynamic system can still be obtained, where the solution spans regions of either positive or negative velocity, and the final conditions for one region become the initial conditions for the following region across the sign change. When the velocity is zero and the applied force is less than the static force, the system is simply static until the necessary force threshold is reached by the motor.

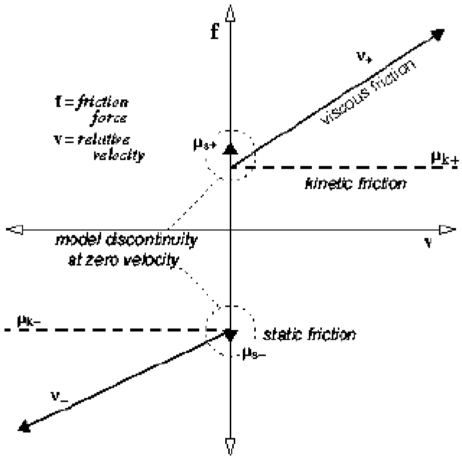

Figure 12. Friction model

To simplify the friction modeling for simulation and controller design purposes, the positive and negative dry friction values are assumed to be equal and opposite, and any discrepancy is treated as a modeling error which is compensated for via some integral control action.

To identify the actual friction in the system, the motor is ramped up slowly from zero force, and the moment the cart begins to move, the applied force is frozen and recorded. This force is thus equivalent to the actual static friction force. After the cart reaches a steady speed the force is next reduced from just above the static friction force value until the cart stops. This is thus the actual kinetic friction force. The viscous friction is estimated by applying a step force to the motor and observing the rise time of the cart velocity.

6. CONCLUSIONS

The mechatronics design of an inverted pendulum system for engineering education is achieved. After applying the dynamic system investigation process on this system, it is well understood and a compact design result is accomplished. The modeling, analysis, and control system design are considered along with experimental validation and the important key issues in hardware implementation. Since we take an educational approach in engineering, the control system design is started from a very conventional way, i.e., classical control design, and the other types of controllers are demonstrated without going into too much details of the theory.

The friction has very important effects on the modeling and control system design such that it causes a drift of the cart to one way in the classical control implementation. For this reason, the static friction is identifed and its effect is reduced on the system by placing a simple friction compensation.

In the future, it is planned to add a swing-up controller so that the pendulum can be erected by making the system unstable in some acceptable range. Then, one of the studied controllers may be switched when the stick is at the upright position and the control objectives addressed in this paper may be accomplished again. Furthermore, some other types of controllers may be designed to make students more familiar with the principles of control system design.

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